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A Simple, Functional Model of Modern Naval Conflict

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WORLD WAR II BROUGHT THE FIRST full flowering of air-sea warfare, active defenses, and the extensive use of electronic sensors and countermeasures. These were revolutionary changes in naval warfare, and they generated a concomitant need to revolutionize the modeling and analysis of conflict. That need was soon filled during, and especially after, the war, as the operations research community grasped the possibilities for their profession of the newly available digital computers. Before long, computer-based conflict models had become the accepted touchstones of prowess in conflict analysis. The extent of present-day reliance on computer-based models, as well as insights into some of the problems they brought with them, are presented well in a book on military modeling published a decade ago.[1]

The evolution of military modeling as a widely practiced profession has, as could be expected, left the non-specialist military thinker, civilian, or serving officer "out of the loop." Whereas such thinkers might have the numerical literacy to understand Bradley A. Fiske's attrition tables or even the facility with simple differential equations needed to read F.W. Lanchester's original work, they would almost certainly have to take the products of modern operational research work on faith alone, if at all.[2]

The purpose of this essay is to rescue the non-specialist in operations research from this dilemma. Its aim is to provide a cognitive tool, accessible to those who

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sense the need for a credible means of quantitatively understanding modern conflict but are not conversant with the arcane ruminations of the specialist. That tool may serve those who want to do their own "back of the envelope" calculations, if only as a "sanity check" on obscurely based results. The reader need have only a reasonable notion of what is meant by probability of success (or alternatively, military effectiveness) for a specified event and be able to read with comprehension simple arithmetical statements expressed in elementary algebra.

In what follows, we will do what should be done for any analytical model to test its credibility, namely:

• Provide a complete construction of the model, including definitions of all terms to be used (i.e., the *vocabulary* of expression) and derivations of all expressions to be used (the *statements* to be made);

• Apply the model to an historical example sufficiently broad to make use of all of the model's principal expressions and thus *empirically verify* its usefulness; and,

• Reconcile the model to self-evident or rigorously proven truths and relationships in order to project the model's validity beyond selected empirical examples.

Construction of the Model

Modern naval conflict can be thought of as comprising in essence three fundamental processes, or functions, potentially to be performed by each of the adversaries in a tactical action. Those functions are:

• target detection and identification;

• attacking the targets; and,

• active defense against attack.

Not all of these functions are always performed by both parties. An example from the Falklands conflict is apt. A salvo of two Exocet missiles was launched by Argentine Super Etendard aircraft against the destroyer HMS *Sheffield*; one of the missiles found its target, and, although its warhead failed to explode, its fuel ignited combustibles, and the ship could not be saved. The Argentine force had performed the first two functions of the list above, and the British ship none. Nonetheless, in a full-blown tactical action each party might perform all three functions, so in a complete functional model of modern conflict they must all be represented, and for each participant.

We will now construct a model using probabilities of success for each of the three fundamental functions as its primary terms, or vocabulary. S_B and S_R will respectively denote the probabilities of success for the Blue and Red forces in the "scouting," or target detection and identification, process. A_B and A_R will denote the probabilities of each offensive attack destroying its targets, *in the absence of any active defense*. That is, in the example of Blue attacking a single Red ship, A_B is the likelihood that the Blue salvo will put the *undefended* Red ship totally out of action, at least for the duration of the fight (that is, a "mission kill"). A_B is thus a measure of the lethality of the BLUE salvo against RED when only RED's *passive* defense characteristics are relevant. Finally, D_B and D_R will respectively denote the probabilities of a successful active defense by each force, whether by "hard" or "soft" kills.* For the terms as now defined, the interactions between BLUE and RED forces in tactical action are expressible in the following two equations, where K_B and K_R denote the respective probabilities of the BLUE and RED forces being destroyed:

$$K_B = S_R A_R (1 - D_B) \quad \text{(Equation 1)}$$

$$K_R = S_B A_B (1 - D_R) \quad \text{(Equation 2)}$$

Since all the symbols used represent probabilities of success, they must represent numerical values lying in the range from zero to one. When they hold the lower-bound value of zero, they represent no likelihood whatever of success; the upper-bound value of one represents certainty of success. For any probability of success, such as D_B, its *ones-complement*, $(1 - D_B)$, represents the corresponding probability of failure. This follows because the process represented—active defense, in this example—must either succeed or fail, and so the sum of the probabilities D_B and $(1 - D_B)$ must equal one, and of course it does. Given all of that, equation (1) can be expressed in words as follows: the probability of RED's success in destroying BLUE equals RED's probability of success at detecting and identifying its targets, multiplied by RED's probability of successful attack (in the absence of active defense by BLUE), and multiplied by BLUE's probability of failure at active defense. Clearly, equation (1) is a far more economical way of making the statement.

The two equations can be simplified a little by substituting the terms V_B for $(1 - D_B)$ and V_R for $(1 - D_R)$, where these new terms denote the probabilities that each side is *vulnerable* due to failure of its active defense. The equations then become,

$$K_B = S_R A_R V_B \quad (3)$$

$$K_R = S_B A_B V_R \quad (4)$$

These expressions are of the simple, multiplied-term form because we are dealing with *conditional* probabilities. So, in equation (4), V_R is the probability that RED's active defense is vulnerable, where BLUE attacks with effectiveness A_B, and BLUE has detected and identified the target with a probability S_B. For illustration, assume that BLUE has a 0.5 probability of finding a RED ship as a target and also

* A "hard" defensive kill involves the physical destruction of the attacker or the weapon—for instance, an incoming cruise missile. A "soft" kill simply denies the attacker a hit—for instance, by electronically deceiving that missile.

a 0.5 probability of effectively attacking that target once it is detected (not considering the target's active defense). Assume further that the RED's active defense has a 0.5 probability of defeating an effective attack (or, equivalently, a 0.5 probability of being vulnerable to such an attack). Then the overall likelihood of BLUE killing this RED ship is (0.5) · (0.5) · (0.5), or 0.125.*

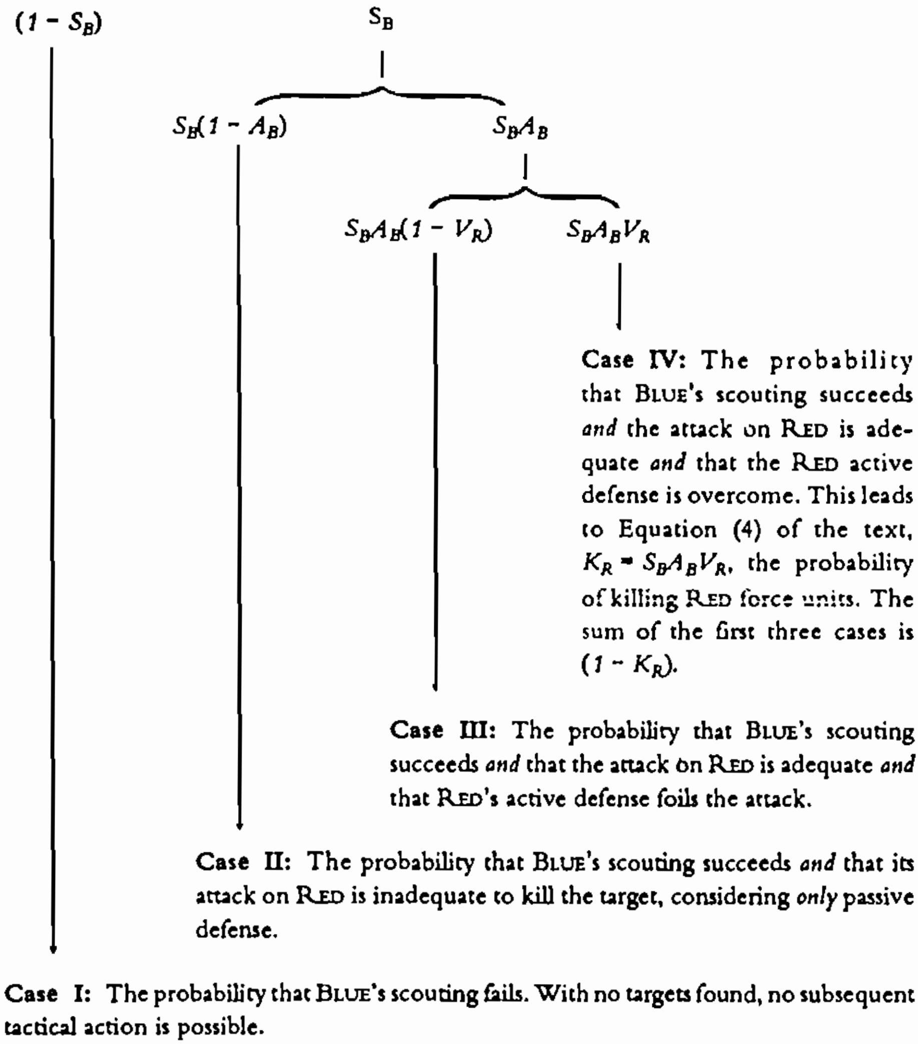
Obviously, estimating conditional probabilities precise enough to draw fine distinctions from equations such as (3) and (4) is a matter for experts. For the rational analysis of modern conflict, however, we will not need precise values, as will later be shown by example. Before proceeding to that, however, a little more work will expand the scope and usefulness of the model much beyond that of equations (3) and (4) alone. Those two equations depict only the probabilities of success for each force—K_B, for example, being the probability of RED's success at "killing" targets in the BLUE force. But failures are often better teachers than successes, and so it is worthwhile to expand the model to account for them.

The best way to explore all possibilities within the scope of our three-function model is with the aid of a taxonomic diagram, figure 1. This diagram identifies all categories of outcomes possible when BLUE wishes to take action against RED; a similar diagram could be constructed for RED against BLUE. Reading horizontally across the top, we consider the two probabilities involved in the scouting process. $(1 - S_B)$ is the probability that BLUE acquires no targets, of which the bottom-line result is Case I, in which nothing of a tactical nature happens. S_B is the probability that BLUE does acquire targets, and in the next horizontal line below it is multiplied by each of the attack-phase probabilities. Completing the top horizontal line, we note that $(1 - S_B)$ plus S_B equals a total probability of 1, as it should.

The first term of the second line is $S_B(1 - A_B)$, the probability that targets are detected and identified *and* that the BLUE attack is ineffective (e.g., all misfires, misses, or duds, without regard to RED's defense). This term leads to Case II, which represents a tactical attempt without result. The second term on this line is $S_B(A_B)$, the probability of successful scouting *and* a potentially lethal attack (i.e., RED's defense not yet considered) by BLUE. The sum of the two terms on this horizontal line equals S_B, as it should. This second term, $S_B(A_B)$, must now be multiplied by each of the active-defense-phase probabilities (here represented in terms of vulnerabilities) on the line below. Those products yield the Case III and Case IV bottom-line results, which are $S_BA_B(1 - V_R)$—the probability that the BLUE attack is foiled by RED's active defense—and $S_BA_BV_R$, the probability that the BLUE attack succeeds. The sum of the two terms on this third horizontal line of the

* The raised dot means "multiplied by." We use it here to avoid possible confusion of the usual multiplication symbol with the familiar variable *x*.

Figure 1
A Taxonomic Outcome Probability Diagram



taxonomic diagram is, properly, S_BA_B. Further, the final expressions, from $(1 - S_B)$ to $S_BA_BV_R$, add up to a probability of one:

$$(1 - S_B) + S_B(1 - A_B) + S_BA_B(1 - V_R) + S_BA_BV_R = 1. \quad (5)$$

That they do proves that we have considered all of the probabilities that can occur, that is, the universe of possibilities that exists for the combinations of scouting and attack by BLUE, confronted by active defense by RED. As will shortly be shown by a well known example, all the bottom-line results of figure 1 can and do occur in the real world of naval tactics.

At this point, the model can be extracted from the realm of operations analysts and probability mathematicians and made more useful to practitioners interested in thinking rationally, systematically, and constructively about modern naval warfare. We may now propose and use some practical approximations.

On a *per target* basis, figure 1 can be readily interpreted and understood for the conditions of modern naval warfare, certainly with respect to the cases that establish the bounds of possibilities. First, considering a Blue attempt against Red, an individual target is either detected and identified or it is not, and so we need only consider the cases of $S_B = 1$ and $S_B = 0$. Second, modern offensive weapons (e.g., missiles, guided bombs) are very lethal in relation to the fragility and volatility of their targets (e.g., ships or aircraft with little passive protection carrying stores of fuels and ordnance). With few (but significant) exceptions, salvos of these weapons, if of appropriate size, are more than a match for their targets.[3] Hence, we may consider only the cases of $A_B = 1$ for a properly delivered and routinely fortunate attack and $A_B = 0$ for all others. Finally, given the above, which implies near-total reliance on active countermeasures, that defense is either essentially perfect, for which $V_R = 0$, or else virtually useless, for which $V_R = 1$.

Using these "all or nothing" simplifying assumptions, the four possible outcomes of figure 1 can be derived as summarized in figure 2. These simplified outcomes are no longer fractional probabilities as in figure 1 but all-or-nothing cases—"no kill," i.e., $K_R = 0$, or "kill," $K_R = 1$—applicable to an *individual target* in the Red force. A similar set of outcomes is easily derived from a Red attempt against Blue.

An Empirical Example

Armed with this simplified version of the more general model, we will now examine the battle of Midway.[4] We will actually consider only one aspect of the battle, treating it as an action in which each side sought to destroy the aircraft carriers of the other side by repeated attacks or attempted attacks, ignoring ancillary events such as the Japanese strike on Midway itself. The analysis is on a per-target (carrier) basis; U.S. forces are denoted as Blue, the Japanese as Red. Each attack wave, or "pulse," is summarized by the expression for the attacker's success probability (which is of course known), given as an illustration of how to employ this probability model.

In addition to the all-or-nothing probability values of 0 or 1 for the three basic tactical processes, we add the additional symbol U, for either "unknown" or "unneeded." U is used whenever an earlier probability of the sequence S_B, A_B, V_R is zero, making the others irrelevant since the product is zero in any case. Thus, if Blue succeeds at scouting but attacks ineffectively, the result can be summarized as a simplified Case II (figure 2), by the statement

Figure 2

Simplified Outcomes

(Single-target, simplified-assumption outcomes for the general cases of figure 1)

Case I

$S_B = 0$ (scouting fails), so the known outcome from figure 1 is $(1 - S_B) = 1$, or certainty, and no other outcomes are possible. The three-process description for simplified Case I is therefore:

$$S_B A_B V_R = 0 \cdot A_B V_R = 0 \text{ (no kill).}$$

Case II

$S_B = 1$ (scouting succeeds) *and* $A_B = 0$ (attack is inadequate), so the known outcome is $S_B(1 - A_B) = 1$, and no other outcomes are possible. The three-process description for simplified Case II is therefore:

$$S_B A_B V_R = 1 \cdot 0 \cdot V_R = 0 \text{ (no kill).}$$

Case III

$S_B = 1$ (scouting succeeds) *and* $A_B = 1$ (the attack is adequate), *but* $V_R = 0$ (RED's active defense is invulnerable), so the known outcome is $S_B A_B(1 - V_R) = 1$, with no other outcomes possible. The three-process description for simplified Case III is:

$$S_B A_B V_R = 1 \cdot 1 \cdot 0 = 0 \text{ (no kill).}$$

Case IV

$S_B = 1$ (scouting succeeds) *and* $A_B = 1$ (the attack is adequate), *and* $V_R = 1$ (RED's active defense fails), so the only possible outcome for simplified Case IV is:

$$S_B A_B V_R = 1 \cdot 1 \cdot 1 = 1 \text{ (a kill).}$$

$K_R = S_B A_B V_R = 1 \cdot 0 \cdot U = 0$. First modelled are Japanese attacks on U.S. carriers.

- First attack on USS *Yorktown* by *Hiryu*-based dive bombers:

$$K_B = S_R A_R V_B = 1 \cdot 1 \cdot 1 = 1$$

(The *Yorktown* was disabled; but progress in quenching fires and patching the flight deck was rapid, and speed was restored to nineteen knots. Nevertheless, she had ordered her returning aircraft to land on *Hornet* or *Enterprise* and did not resume flight operations during the battle, so this attack must be credited to RED as a "mission kill.")

- Second attack on *Yorktown* by *Hiryu*-based torpedo planes:

$$K_B = S_R A_R V_B = 0 \cdot U \cdot U = 0$$

(An interesting situation: the Japanese had correctly assessed that the *Yorktown* was out of the fight and reattacked only in the mistaken impression [produced by the remarkably fast repairs] that they had found some different carrier. Hence, with regard to the attack *intended*, $S_R = 0$, and the "added value" to their score of carriers killed is 0.)

Now follow a number of U.S. attacks on Japanese carriers.

- On *Hiryu*, by Midway-based torpedo planes:

$$K_R = S_B A_B V_R = 1 \cdot 1 \cdot 0 = 0$$

(Active defense—by Zero fighters, evasive maneuvers, and machine-gunning of torpedoes—was perfect.)

- On *Akagi*, by Midway-based torpedo planes:

$$K_R = S_B A_B V_R = 1 \cdot 1 \cdot 0 = 0$$

(Active defense by Zero fighters and evasive maneuvers was perfect.)

- On *Hiryu*, by Midway-based glide-bombing planes:

$$K_R = S_B A_B V_R = 1 \cdot 0 \cdot U = 0$$

(Five of the planes survived the Japanese active defense and released bombs, with no hits. The attack technique was ineffective, and $A_B = 0$.)

- On *Kaga*, by Midway-based glide-bombing planes:

$$K_R = S_B A_B V_R = 1 \cdot 0 \cdot U = 0$$

(Three planes survived to release bombs, with the same conclusion as in the attack above.)

• On *Soryu*, by Midway-based B-17s in high-level bombing attack:

$$K_R = S_B A_B V_R = 1 \cdot 0 \cdot U = 0$$

(Gravity-bombing the moving ship from 20,000 feet was totally ineffective; $A_B = 0$.)

• On *Hiryu*, by Midway-based B-17s in high level bombing attack:

$$K_R = S_B A_B V_R = 1 \cdot 0 \cdot U = 0$$

(Same as the attack on *Soryu*, above.)

• Attempted attack by Midway-based, glide-bombing aircraft on a carrier, delivered instead against battleship *Haruna*:

$$K_R = S_B A_B V_R = 1 \cdot 1 \cdot 0 = 0$$

(With respect to the hoped-for attack on a carrier, the intensity of the active defense over the whole formation must be credited as $V_R = 0$. If we were considering battleships as targets, the score would be the same, a combination of fighter defense and *Haruna*'s evasive maneuvers. Here we grant that a glide-bombing attack might have been effective [$A_B = 1$], despite the two earlier examples.)

• Attack on *Soryu* by *Homet*-based torpedo planes:

$$K_R = S_B A_B V_R = 1 \cdot 1 \cdot 0 = 0$$

(All fifteen planes of the attacking force were shot down by Zeros.)

• On *Kaga*, by *Enterprise*-based torpedo planes:

$$K_R = S_B A_B V_R = 1 \cdot 1 \cdot 0 = 0$$

(Active defense by Zeros and *Kaga*'s evasive maneuvers.)

• Sortie by *Homet*-based dive bombers:

$$K_R = S_B A_B V_R = 0 \cdot U \cdot U = 0$$

(These thirty-five planes found no targets.)

• Attack on *Hiryu* by *Yorktown*-based torpedo planes:

$$K_R = S_B A_B V_R = 1 \cdot 1 \cdot 0 = 0$$

(Defense by Zeros plus evasive maneuvers by *Hiryu*.)

• On *Kaga*, by *Enterprise*-based dive bombers:

$$K_R = S_B A_B V_R = 1 \cdot 1 \cdot 1 = 1$$

(With active defense unready, at least four hits were made, with secondary explosions of fuel and ordnance stores.)

• On *Akagi*, by *Enterprise*-based dive bombers:

$$K_R = S_B A_B V_R = 1 \cdot 1 \cdot 1 = 1$$

(At least three hits, plus secondary explosions.)

• On *Soryu*, by *Yorktown*-based dive bombers:

$$K_R = S_B A_B V_R = 1 \cdot 1 \cdot 1 = 1$$

(Again, at least three hits, plus secondary explosions.)

• Finally, an attack on *Hiryu* by *Enterprise*-launched dive bombers (some from the now-disabled *Yorktown*):

$$K_R = S_B A_B V_R = 1 \cdot 1 \cdot 1 = 1$$

(Probably four hits.)

A disturbing lesson arises from the U.S. performance in these engagements, as summarized in tables 1 and 2. The mere 6.67 percent of Case I outcomes testifies to the quality of U.S. pre-battle intelligence and the scouting based upon it. However, the Case IV outcomes add up only to what amounts to a random draw, and they reflect no credit whatever on U.S. preparation and training for battle, whereas the Case III percentage testifies to the excellence of Japanese active defense. In passing, we may note that the simplified probability model was able to handle all of the fifteen attempted attacks by U.S. forces. The table 2 percentages, expressed as decimal fractions and used with the case-outcome expressions of figure 1, can be used to calculate the three fundamental process probabilities of success, in an after-the-fact sense, for the battle of Midway as modeled here. The answers are, $S_B = 0.933$, $A_B = 0.714$, and $V_R = 0.400$; and $S_B A_B V_R$ is therefore 0.266,* agreeing

* The slight discrepancy is due to calculation with only three significant figures.

Table 1
Per-Target Actions
Attempted by U.S. Forces (Simplified Cases)

Against *Kaga*	Against *Akagi*	Against *Hiryu*	Against *Soryu*	Against Indeterminate
II	III	III	II	I
III	IV	II	III	III
IV		II	IV	
		III		
		IV		

Table 2
Attempted U.S. Attacks

Simplifed Cases	Total Number	Percentages
I	1	6.67
II	4	26.67
III	6	40.00
IV	4	26.67

Note: A random drawing for each species of outcome would yield 25 percent each.

with the table 2 value for the fraction of all U.S. sorties that were successful at Midway. Thus, summarizing *per-target* results gives consistent *battle* results, as it should, and permits the estimation of probabilities of success for the two U.S. offensive functions and the Japanese active defense vulnerability (or, alternatively, the Japanese active-defense success probability, 0.600).

The reader should understand that these numbers are an *example* set—the product of the set of assumptions used together with facts from which they were obtained. It is *fact* that four Japanese carriers were destroyed, that fifteen sorties were flown against them of which fourteen resulted in actual attempted attacks, and that the four kills were achieved by dive bombers. But it was *assumed* that five torpedo-bomber and one glide-bomber attack would have been equally effective had they not been foiled by the Japanese defense. That assumption seems reasonable (at least for torpedo attacks) in light of World War II actions other than Midway. Therefore, the numbers $A_B = 0.714$ and $V_R = 0.400$ are derived from a mixture of facts and assumptions, with only $S_B = 0.933$ being indisputably factual. So what we have here is not "irrefutable truth" even for Midway, where we know the bottom line. Rather, it is an example of how to use a rigorous, conceptual framework to organize analysis wherein many numbers affect the result, and many numbers may be debatable.

Reconciliations to Other Theory

Having shown the model's potential usefulness with the empirical example of the battle of Midway, we may now buttress its credibility in a more general sense. First, we can show that the model is not inconsistent with Frederick William Lanchester's well known "*n*-square law." The ratio of equation (4) to equation (3) is:

$$\frac{K_R}{K_B} = \frac{S_B A_B V_R}{S_R A_R V_B}. \tag{6}$$

Lanchester's original work defined forces as having equal "fighting strengths" when each was capable of reducing the opposing force in the same proportion per unit of time. For example, if a force of five ships fights ten ships, and the larger force loses two ships per hour while the smaller loses one per hour, they are said to be equal in fighting strength. In the terminology of the present model, equivalence in fighting strength exists if each force has the same probability of destroying the other in any specified time interval—such as during an entire engagement. Thus, K_R and K_B must be equal, so that their ratio, as in equation (6), would be one.

Also, Lanchester's work did not address the subject of scouting but merely assumed that the forces in conflict would detect and identify all targets—i.e., in the present model, the situation of $S_B = 1$ and $S_R = 1$. Similarly, Lanchester did not contemplate active defense but analyzed only the attrition between forces taking offensive actions; the analogous situation for the present model is $V_B = 1$ and $V_R = 1$. Thus, in the Lanchester approach equation (6) degenerates to:

$$\frac{A_B}{A_R} = 1. \tag{7}$$

Now let n_B and n_R denote respectively the numbers of fighting units in the Blue and Red forces, and f_{KR} denote the probability of a single Blue unit killing a Red unit during a specified time interval. Then the *expected* number of Red units killed is $n_B f_{KR}$, and the *fraction* of the whole RED force killed is given by $n_B f_{KR}/n_R$. However, under the assumption that the probability of success in killing the whole RED force applies as well to each of its individual units, the expected value for RED units killed is $n_R A_B$, and the *fraction* of the total force killed is $n_R A_B/n_R = A_B$. But we have just derived another expression for this fraction, and so we can write:

$$A_B = \frac{n_B f_{KR}}{n_R} \tag{8}$$

and, by analogy,

$$A_R = \frac{n_R f_{KB}}{n_B}. \tag{9}$$

By substituting (8) and (9) into (7) and rearranging, it may be found that

$$f_{KB} n_R^2 = f_{KR} n_B^2. \tag{10}$$

Except for different notation, this equation is precisely Lanchester's *n*-square law.[5] Thus, not only is the present model consistent with the Lanchester law, but the latter is derived from a special, reduced case of the former, expressed as equation (7).

One more special case is of interest, to validate the model further. Consider the situation where RED completely surprises BLUE; that is, BLUE is able neither to defend actively nor return offensive fire. The corresponding applications of equation (3) and (4) are:

$$K_B = S_R A_R V_B = S_R \cdot A_R \cdot 1 = S_R A_R \tag{11}$$

and,

$$K_R = S_B A_B V_R = S_B \cdot 0 \cdot U = 0, \tag{12}$$

where we have contemplated that BLUE might have actually detected and identified its potential targets, but not in time to do anything about it. RED survives intact, since $K_R = 0$, whereas BLUE suffers the full effect of RED's assault. In the very simple example cited earlier of HMS *Sheffield* (taken as BLUE), the version of equation (11) was:

$$K_B = S_R A_R V_B = 1 \cdot 1 \cdot 1 = 1. \tag{13}$$

It is important that a model be able to depict surprise in conflict, since surprise is the best means available to a tactical commander for degrading the enemy's capabilities.

Finally, a model worthy of general use should be self-evidently true to anyone conversant with its idiom. The present model has that property, as is evident from a rearrangement of equation (6):

$$\frac{K_R}{K_B} = \left[\frac{S_B A_B}{V_B}\right] \Big/ \left[\frac{S_R A_R}{V_R}\right] \qquad (14)$$

Expanded into words, this equation says that BLUE can maximize his ratio of kills of RED to his own losses by maximizing the effectiveness of scouting and attacking, and by minimizing his own vulnerability—no more than a truism, as it should be. Its essence was captured in similar words by Mao Tse-tung:

The first law of war is to preserve ourselves and destroy the enemy.[6]

A model that gives quantitative meaning to such simple words can be a powerful cognitive asset to the analysis and understanding of modern naval conflict.

Notes

1. Wayne P. Hughes, Jr., ed., *Military Modeling* (Alexandria, Va.: Military Operations Research Society, Inc., 1984). The introduction provides a good summary.

2. Bradley A. Fiske, *The Navy as a Fighting Machine* (New York: Charles Scribner, 1916), p. 284. Fiske published a number of attrition tables, over a number of years; this reference is representative. For Lanchester, J.R. Newman, *The World of Mathematics* (New York: Simon and Schuster, 1956), pp. 2138–57. This republication of Lanchester's original work is available in most modern technical libraries.

3. The exceptions are certain vessels of such size and internal arrangement that they may well remain operational even after repeated hits. Modern aircraft carriers are so elaborately equipped against conflagration that, their obvious vulnerabilities notwithstanding, disablement is at least not to be assumed. Also, as both Iranian and Iraqi pilots learned in the "tanker war," however many Exocets one launches, it can be surprisingly difficult even to set fire to bulk oil carriers—seemingly the most helpless and combustible of targets. Notwithstanding, there are antiship weapons (notably the SS-N-19 cruise missile) powerful enough at least to put out of action all but the largest and best protected target, and the vast majority of warships are not in that category. One Exocet destroyed the *Sheffield*, two of them wrecked the frigate USS *Stark* (FFG 31), and Japanese bombs and torpedoes wreaked havoc at Pearl Harbor among even heavily armored ships not putting up an effective active defense.

4. The following discussion is based upon Walter Lord, *Incredible Victory* (New York: Simon and Schuster, 1968). Two other classics on the subject also proved helpful: Mitsuo Fuchida and Masatake Okumiya, *Midway, the Battle that Doomed Japan* (Annapolis, Md.: U.S. Naval Institute Press, 1955), and Gordon W. Prange, *Miracle at Midway* (New York: Penguin, 1983).

5. As Lanchester expressed it, his *n*-square law is:

$$Nr^2 = Mb^2,$$

where r and b denote respectively the numbers of RED and BLUE fighting units, and N and M the "fighting values" of individual units.

6. Robert Debs Heinl, Jr., *Dictionary of Military and Naval Quotations* (Annapolis, Md.: Naval Institute Press, 1966), p. 169.

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