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A Method of Conceptualizing Combat Theory

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To the military professional, the principles of combat must be more precisely defined than simply lists of elements such as surprise, mobility, concentration, security, and economy of force. What is needed in order to understand the full implications of high technology on combat is a language that is partly written and partly symbolic.

A METHOD FOR CONCEPTUALIZING COMBAT THEORY

Adapted from a lecture

by

Professor Chantee Lewis

Weapons, mobility, and surveillance possibilities are key controllable elements which govern the evolution of combat. The difference between ancient arms and their technology and warfare today is so great that the combat lessons from the past have only a limited transfer value to current situations. In Nelson's day maneuvering allowed the more skillful side to bring superior weapon power to bear, so that many ships could concentrate on a few of the enemy. The advent of steam, followed by aircraft, nuclear power and standoff weapons has reduced the relative importance of maneuvers. Surprise maneuvers are, of course, still valuable. Admiral Scheer's surprise countermarch at Jutland or the Japanese assault at Pearl Harbor are classic examples of surprise, be it tactical or strategic. Coordinated efforts that take advantage of modern technology, such as overhead satellite surveillance and the unique firepower of lasers, can produce future surprises.

Much of what has been recorded on combat theory has been limited to one medium of communication, the written word. Several writers, including Clausewitz and Douhet, have likened combat to the science of mathematics, but they did not elaborate to any extent on this point. One is then faced with the obvious question: Is the written word enough?

Assuming that professional naval officers have absorbed the written descriptions of combat theory expressed in the so-called principles of war (the objective, surprise, mass, firepower, speed, et cetera), is there another means of communication which might give them a different *lens* through which to view the science of war? The military professional needs a hybrid language, one that is partly written and partly symbolic, in order to understand better the complexities brought about by applying advanced technology to combat. Such a language can help to conceptualize such a complex subject as large-scale high technology combat. One such hybrid language worth considering is a quantitative and graphic analysis of combat.

Figure 1 graphically illustrates this point. Starting with a combat problem or objective, then by adding various inputs, it is possible to apply various analogies and the principles of war in order to make the best strategic and tactical decisions. However, in today's complex situations this approach is too limited. Intuitive judgment must have feedback by what can be called the quantitative process model.

Clausewitz is right: there is a "fog of war" that has yet to be penetrated. An analytical approach, in addition to verbal descriptions, can cautiously probe the boundaries of the unknown and can clarify some important combat factors. This is not war by the numbers but the use of numbers to assist a commander in the conduct of operations.

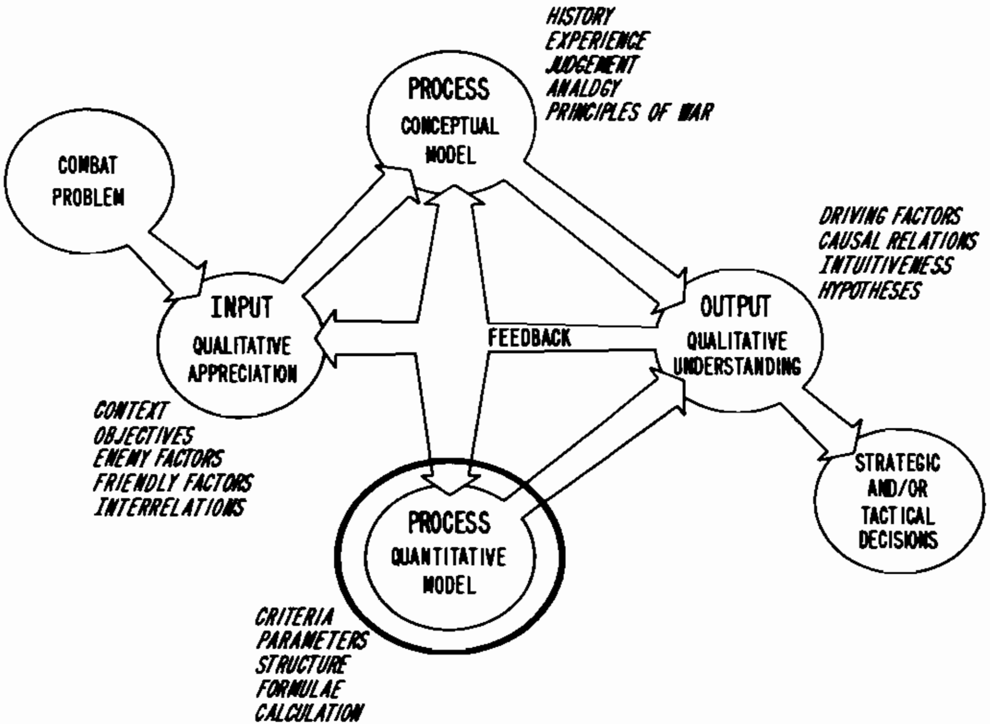
The quantitative understanding of combat is accomplished by a number of methodologies. At the micro level, it may only be a short probability statement for understanding the expected impact of salvos from a naval gun for certain firing envelopes. At the macro level it could be a complex many-on-many model of incorporating a wide range of disciplines to determine or to analyze the total campaign. At present, a number of methodologies or procedures have been developed that help us to understand the key variables in combat at several levels of a force engagement. Thus, a one-on-one engagement analysis can be a building block for battle analysis.

Symbolic combat theory is not based upon any axioms comparable to F = MA. It starts, usually, with a hypothesis (e.g., effective firepower is linearly related to the level of technology) or a model that is to be tested in field exercises and compared with combat results. Then, if the real world data does not reject the model, it is tentatively accepted as an explanation, in part, of what happens in certain situations. Thus, we build and develop on quantitative theory of combat by a "cut and try" pragmatic method.

Over the past 30 years, this method of trying to understand combat has attracted increasing attention both at home and abroad. The Soviets, with their long-term interest in mathematical relationships, have an active analytic program which seeks to verify combat theory. This, of course, does not mean that we should also have such an effort. However, it is a methodology that may, as a bonus, help to understand with greater clarity the general principles that affect not only ourselves but also the thinking of our potential opponents. A way to classify and look at the "rich" range of quantitative viewpoints of combat is to use various combat models. Figure 2 represents a hierarchy of such models.

Each of these models has its strong points and its place in the understanding of combat. Tabletop, one-on-one analysis is useful as a building block where a specific threat can be played against a weapons system under consideration. The players gain insight into which variables drive the situation for specific threats and types of technology.

Deterministic models can be thought of as simple abstract concepts—such as the weapons effectiveness equation or the sonar equation. All deterministic models yield only closed form solutions. Computer aided models expand on the closed form models and can handle the random process of combat. They can be dynamic and self-correcting. Computer games or machine-simulation programs, if properly programed, can simulate conflict in accordance with detailed threats and rules of engagement for various levels of inputs.



Source: Henry Young, Consultant

Fig. 1—Complementary Relation Between Qualitative and Quantitative Models

COMPLEXITY ⟶

Increase in Hieratical Order ↓	One-On-One	Many-On-Many	Campaign Analysis
Table Top Games	Yes	?	No
Deterministic Games	Yes	Limited	No
Computer Aided Models	Yes	Yes	?
Computer Games	?	Yes	Limited
Mixed Simulation Models	No	Yes	Yes
Field Exercise	No	Yes	Yes
↓ Combat			

Note: Yes indicates the model can simulate the degree of complexity. No indicates it cannot handle the complexity. A question mark indicates the outcome is in doubt.

Fig.2—Applications of Hierarchy of Models vs. Complexity

As an input, computer games use basic analytical models with closed form subroutines. Due to their high speed of play and ease of replication, computer models are not only useful as training devices but are also powerful tactical and analytical research tools. Computer games attempt to answer "what if" questions and indicate how various outcomes would change if the assumptions are varied. Mix-simulation models integrate the strong points of the reasonably understood deterministic models with the man-machine interface.

War gaming at the Naval War College permits the games to have an interjection of the human thought process at each step, which does much to develop insights. For this reason, it has a unique training value to the players. Reasonable simulation of the most complex situations such as air engagements cannot be duplicated by a computer at the present level of technology without a man in the loop. Field exercises, structured or free-play, have a great potential to validate the other models or to show where the theory of the other models fails to follow the real world.

What is the point of all this, if the goal is how to conceptualize combat theory? It is necessary to understand cause and effect relationships and, then, to apply the results of war games to the real world. In an analytical war game, what does it mean to a commanding officer to discover that the chances of success for a particular tactic are 8/10? Does it do him any good to know that on the average he can expect success 8 times out of 10, when in combat he will only have one try?* Perhaps the best that can be expected from war games and theory is a comparative answer rather than an absolute one. Comparative solutions should be useful even if they only show that the probability of success is relatively better with tactic A than with tactic B. At least, for today's technology, it should indicate what fire doctrine or search procedure one would not want to use.

The heart of modeling, war gaming, or analyzing field exercises is derived from a few probabilistic or analytical concepts. For today's technology and total environment of water, land, air, and space, the models listed in figure 2 appear to be greatly influenced by the following analytical building blocks:

*Symbolic models usually handle the critical element of time in but a limited fashion. Such issues as How long will the war last? Is this fight the last battle of war? or Must my same forces fight again tomorrow? are best handled by the exercise of judgment and experience.

- Detection Models (Search Theory)
- Attack/Penetration Models
- Mass vs. Technology Models—the high-low mix models
- Decision Models or Game Theory

Detection or Search Theory in today's naval environment may be the most important concept in understanding dynamic naval conflicts. From the one-on-one to the many-on-many situation, proper use of search sensors in order to detect the enemy first (with sufficient confidence to open fire at the best time) appears, on the average, to determine who kills whom. Long-range, over-the horizon weapon systems (Harpoon, Styx) further emphasize the importance of search theory. For the physical characteristics of sensors (eyes, radar, or sonar), search theory helps to indicate when we should be passive in our search, when our search should be active, and when it should be continuous or in glimpses and what the relationship will be between false alarms and probability of detection. Understanding the tactical use of screens, barriers, or deception devices is closely related to search theory.

After detection has been made or avoided, an attack or penetration might be the next appropriate tactic. Questions of when saturation might occur and the value of salvo firepower as opposed to a single shot can be better understood by looking at basic probability models. As an example, let us assume the enemy can recover from two or less hits per ship and the probability of a single missile penetrating his defense is 0.16. If there is a choice of conducting a salvo attack with 6 missiles, 10 missiles, 20 missiles, or 30 missiles against the one target, what is the probability of two or more hits on the ship? Probability models should not be accepted on faith! Are the events independent? Usually they are not. Is the distribution normal?[1]

Skipping for a moment the third analytic building block, the fourth item (decision models or game theory) can be the heart of the analytical tactical situation. Game theory gives insight into the interaction of our alternatives with the possible enemy courses of action. In game theory the action of each player influences the outcome of certain events. If for each possible event, the algebraic sum of gains and losses to all players is zero—the game is called a zero sum game. In other words, what one gains, the other loses. Of course, many combat situations are non zero; both sides can lose to varying degrees. Knowledge and experience of this theory enable a commanding officer to predict, with some accuracy, the reactions he can expect of an opponent for different tactics, if he pursues him and *if* the opponent uses a similar value system! For example, if an enemy ship with five defense tactics (A, B, C, D, E) were to be attacked by one of four attack choices (1, 2, 3, 4), and if the relative payoffs were as follows, a decision matrix might look like:

		Enemy Defense Choices				
		A	B	C	D	E
	1	10	3	0	6	7
Attack	2	3	5	6	8	10
Choices	3	9	6	7	8	8
	4	2	4	5	1	3

What is the best choice for a conservative approach? Answer: Attack using alternative 3 with an expected payoff of six, even if the enemy did his very best (alternative B).[2]

Game theory clearly points up the resulting conservatism of military doctrine if based upon estimates of enemy capabilities. The crux of the matter is our scale of relative values, indicating the importance of knowing a potential enemy. In addition, the format (matrix) of game theory makes the tactical commander think hard about and gather data on all of the various possible outcomes, in order to assure a consistently better estimate of the situation.

If a commander cannot formulate the matrix of opposing tactics for the situation, he is not properly prepared to make a complex decision. Without the matrix, he might overlook a proper alternative.

Now let us return to the third analytical building block—mass vs. technology models—which is often misunderstood or overlooked by many tactical decisionmakers. This block helps us to understand how much 10 fighters or destroyers are worth (at the point of contact) relative to two groups of 5 fighters or ships at two different points. Understanding the relationship between mass and technology should permit us to do more with the same resources.

Does the combat output of mass relate to the linear relationship of the numbers involved or to some exponential or square number of the mass of inputs? In today's technology the mass may relate more to the number and geometry of the missiles (such as Harpoon, Styx, or Condor) than just the number of launching platforms. This is just another form of the "high"-"low" mix question. It is the principle of concentration, or Lanchester's Laws.

Naval officers will find Lanchester's ideas useful because they provide insights necessary to answer such questions as:

- How do force levels affect tactical allocation issues?
- Do relative target priorities change over time during the battle?
- How does the number of targets affect the optimum attack choice or fire doctrine?
- How do the circumstances of the *start* or *termination* of conflict, under various rules of engagement or engagement policies, affect the possible allocation policies?

This concept addresses the question of under what circumstances might a smaller force (for example, ships, aircraft, or tanks) be expected to defeat a larger force. Can a mathematical measure be assigned to concentrations of firepower, and if so, can such measures appear to describe what actually happens under combat situations?

Frederick William Lanchester, engineer, mathematician, and infantry officer, published many books and technical articles on a wide range of issues. He died in 1946 at the age of 78. His N-square law of the relative fighting strength of two forces is relatively simple, but its full implications are not. Actually, Lanchester's concepts were earlier discussed in general terms by a U.S. naval officer, Rear Adm. Bradley A. Fiske of the Class of 1874.

Specifically, Lanchester first observed that a machinegun (which is technically superior to the rifle for some missions) in his day and age had about a 16 to 1 technical superiority; meaning that 16 effective bullets were put on the target for each effective rifle bullet. In other words, the machinegun could do the work of about 16 riflemen in some situations and the linear law appeared to apply.

If the relative technology between two weapons systems can be determined, then their relative firepower is a product of technology x the number of units, or:

Firepower $\cong$ T x N (the linear law)

Then the firepower of one rifleman = 1x1 = 1 and the firepower of one machinegun = 16x1 = 16.

Lanchester also observed that for average terrain and with average leadership while 16 riflemen could usually overcome a machinegun, approximately 4 riflemen were an equal match for the 16-fold increase in firepower of the machinegun. In World War I there was a 50-50 chance that four riflemen would take a machinegun position. Thus, the two might be considered equally effective. The result was the so-called square law.

A simplified treatment of Lanchester's theoreum goes as follows:

Suppose F units of one force of technology firepower α are engaged by R units of an enemy of technology level β. Suppose further that the combat is such a kind that the firepower of force F is directed *equally* against all units of R and vice versa (no units held in reserve); then the rate of loss between the two forces would be indicated by

$$\frac{dF}{dt} = -k\beta R$$

and $\frac{dR}{dt} = -k\alpha F$; where k is a constant.

If the relative strength of the two forces is equal when their fractional losses are equal then:

$$\frac{1}{\alpha F}\frac{dF}{dt} = \frac{1}{\beta R}\frac{dR}{dt}$$

and from the above by calculus we get $\alpha F^2 = \beta R^2$ or the square law where firepower = $(\text{mass})^2$ X technology

Another example of how a consideration of the mass of the machinegun versus the infantryman might answer an important question is the World War I British query: "Can we reduce the infantryman battalion by 15/16 to save money and be equally well off?" Or, if the technology and firepower shift is 16 to 1, why not reduce the number of troops by 15 every time we deploy a machinegun? Lanchester is thought to have said, "No, it is not quite that simple." If the riflemen in a battalion were to be reduced while the firepower remained constant, the solution might be better expressed by:

$$(\text{Rifleman})^2 \times 1 = (\text{Machine gunner})^2 \times 16$$
$$(1000)^2 \times 1 = (N)^2 \times 16$$
$$N = \sqrt{\frac{(1000)^2}{16}} = \frac{1000}{4} = 250$$

Now what does this mean to naval aircraft, ships, or say Marine Corps tanks? Work by Lanchester, Weiss, and Taylor and by Deitchman and Schaffer on jungle warfare indicate that in a mobile situation, where the sensors can identify specific targets and where it is possible to concentrate firepower on specific targets, the mass has some interesting relationships relative to technology.

The conclusion of an Air Force study on loss rates from all service for air-to-air engagements during World War II and Korea graphically looks like figures 3 and 4.

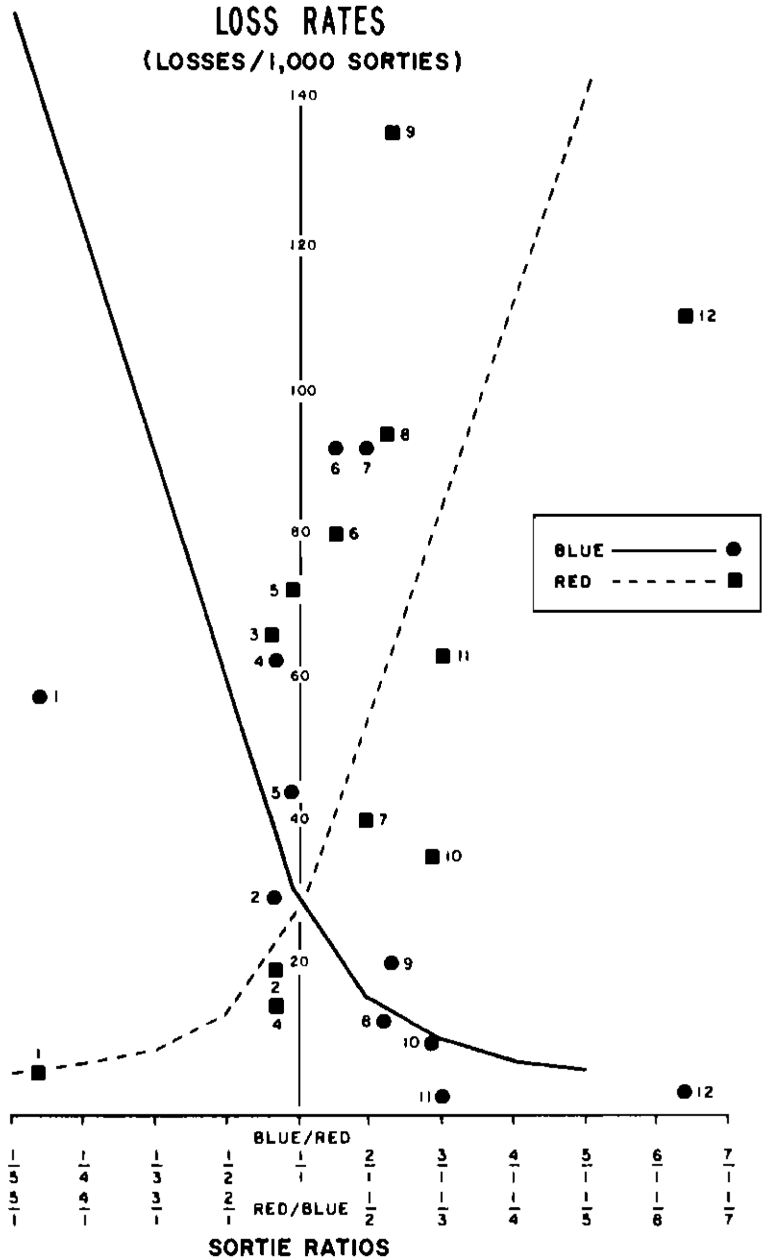
These figures show the losses (aircraft lost per 1,000 combat sorties) on the vertical axis and the relative strength of our forces (Blue) to the Axis of North Korea (Red) forces on the horizontal axis. On the right-hand side of the figures, Blue always had superiority (2 to 1 to 5 to 1) and the relative loss rates favored were those indicated by the slope of the regressive lines. If Lanchester had been exactly "correct," the exponential slope would have been 2 (the square law). This emperical evidence clearly shows the relationship is far from linear, and by regression analyses it is estimated to have an exponential value ranging from 1.4 to 1.8. To illustrate the possible value of fighter "mass" in a square law situation, let us consider how F-4's might be concentrated.

Let us say the enemy has 50 F-4's and we have 70 F-4's with the surge option equivalent to 85 for a short period of time. How much is the surge and/or concentration worth (both sides equal in technology and training)? This example shows *just* the *value* of *mass* without looking at the technology shifts that might be possible. First, a stopping rule is needed. Let us assume that the enemy will withdraw from the field for the time being when we have air superiority and when his losses are equal to or greater than 40 percent of his force (when he loses 20 or his 50 aircraft).

Using the "square law," 70 F-4's vs. 50 would be expected, on average, to result in the following:

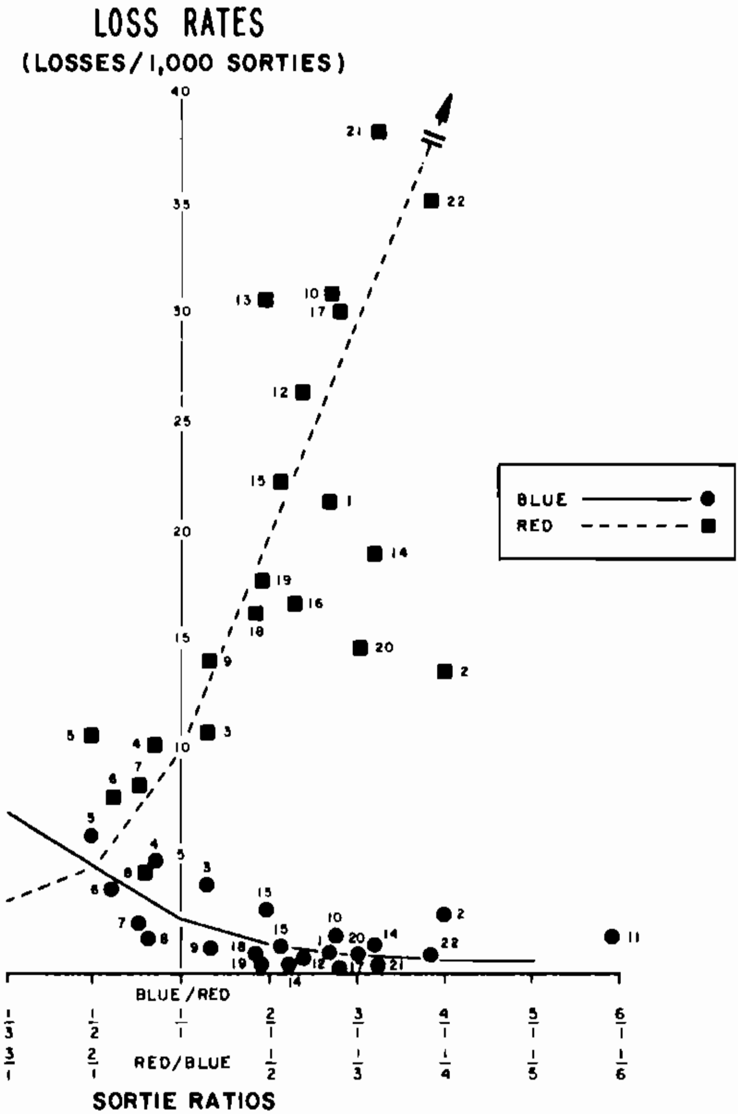
$$(70)^2 - (N)^2 = (50)^2 - (30)^2 \; [40\% \text{ loss}]$$
$$N = 57.45$$

or the kill ratio is $\frac{20}{12.55} = 1.59$



Source: USAF Studies & Analyses, *The Relationship Between Sorties Ratios and Loss Ratio for Air-to-Air Engagements During World War II and Korea* (Washington, D.C.: September 1970), p. 6. Internal unclassified USAF report.

Fig. 3—World War II Loss Rates vs. Sortie Ratios



Source: USAF Studies & Analyses, *The Relationship Between Sorties Ratios and Loss Ratio for Air-to-Air Engagements During World War II and Korea* (Washington, D.C.: September 1970), p. 12. Internal unclassified USAF report.

Fig. 4—Korea Loss Rates vs. Sortie Ratios

Then, if we surged to 85 "ready" aircraft, the relative situation would shift toward:

$$(85)^2 - (N)^2 = (50)^2 - (30)^2$$
$$N = 75$$

or the kill ratio is $\frac{20}{10} = 2.0$, or we might save up to 2.55 F-4's while accomplishing our air superiority mission.

Next, let us examine the high-low mix question. How many F-4's might be equivalent to one F-14 in a fleet defense role? Even if we did not want to choose F-4's, how many F-15's or F-16's are about equal to an F-14 for some missions? In other words, how many low-technology fighters might be traded for one high-technology fighter?

To start the solution, we need a "mass" comparative basis between F-4's and F-14's. One way is to consider the relative costs between the two aircraft, which is about 4:1. In other words, a $400 million procurement air defense budget might buy 100 F-4's or 25 F-14's. Recall figures 3 and 4 and the basic Lanchester square law. The exponential for the "square" law could range from 1.4 to 1.8 to 2.0, depending on which data are used. From this, the relative answer would appear to be indicated by the following:

F-4 F-14

$$(1)\ (4)^2 \approx (T)\ (1)^2$$
$T_1 \approx 16.0$, for an exponential value of 2.0

but

$T_1' \approx 12.13$, for an exponential value of 1.8

and

$T_1'' \approx 6.96$, for an exponential value of 1.4

This means that, depending on the scenario and rules of engagement, if we feel one F-14 is always better than 16 F-4's, then we would want *only* the high-technology aircraft. If for our rules of engagement and mission, the F-14 is not worth at least 6.96 F-4's, then we should retain our low fighter mix. If the answer is estimated to be between 6.9 and 16.0, then we would want a mix of both types of fighters.

Do Lanchester's concepts apply to ASW? World War II data reported by Morse and Kimball and subsequent fleet experience confirm common sense that high value ship sinkings by submarines were directly proportional to the quantity and quality of the enemy submarines. In addition, those sinkings were inversely proportional to the ASW technology and the number of the ASW escorts.

Further, the number of submarines, on average, destroyed by the ASW force is directly related to the number of submarines in the area and the number of escort units. Thus the number of submarines destroyed in each attack is related to ASW technology times mass. Folding these two concepts together produces an interesting exchange ratio that is mainly driven by the "mass" of the ASW force.

In shorthand mathematic notation we have:

$$\text{High value ship losses} \cong k_1 \frac{(N)}{(E)}$$
$$\text{Submarines destroyed} \cong k_2\ (NE)$$

or

$$\text{Exchange Ratio} = \frac{\text{Sub kills}}{\text{H.V. kills}} \cong k_i\ (E)^2$$

where N = No. of submarines
E = No. of ASW units
k_i = relative technology between platforms

Note that we again have a "square" relationship pertaining to the mass or number of ASW units and a linear relationship relating to the technology ratio between submarines and the ASW force.

A simplistic ASW problem will show the possible relative advantage of ASW "mass." Assume that we have 20 escort ships of a specific type to protect 5 high-value task groups that require, or appear to require, surface ASW protection. Let us further assume a standardized set of tactics and a normalized enemy doctrine. How best can we employ the escorts? If we divided them equally, the expected exchange ratio for the threat might look like:

$$\text{Equal force exchange ratio} \cong k_i(5)(4)^2 = k_i(80) \quad (4, 4, 4, 4, 4)$$

An unequal distribution of the escort assets gives us:

$$\text{Nonequal force exchange ratio} \cong k_i\,[3(2)^2 + 2(7)^2] \quad (\text{say } 2, 2, 2, 7, 4) \cong k_i\,(110)$$

In this case, to be successful we need *cover and deception* along with the *careful use of flexible ASW sonar tactics* because the submarine *must perceive each of the high-value task groups equally as a likely target.* Then the submarine is equally apt to engage any one, rather than concentrating on, say the 2, 2, 2 escort groups and letting the 7, 7 groups go. All other factors being equal, the relative exchange ratio appears to be favorable when we use a nonequal distribution of our assets and concentrate our forces.

Of course, this model is simplistic and it must be used with caution. Without balancing judgement and experience, we might conclude that putting all escorts on *one* convoy and letting the other high value ship groups go unescorted would be best. This would be foolish! It extrapolates the model too far and assumes perfect cover and deception. But the model does indicate that, within the ranges observed in World War II and in recent fleet exercises, unequal distribution of forces (with flexible cover and deception) is superior to equal distribution.

In summary, the principles of combat must be more precisely defined than just lists of elements such as surprise, mobility, concentration, security, and economy of force. What is needed in order to understand the full implications of high technology on combat is a hybrid language that is partly written and partly symbolic. The writings of Mahan, Corbett, and Clausewitz are starting points. But let us also give a place of importance to Blackett, Morse, and Kimball and, last but not least, to Lanchester.[3] The pitfalls of oversimplification are many and our responsibilities are great when we attempt to establish a relationship between combat outcome and the various possible inputs. In no way is it contended that such inputs directly determine the output. But such inputs as "mass" do influence the outcome in some recognizable, measurable form, and the use of "mass" while evaluating what the enemy could do (game theory) permits doing more for the same amount of resources.

Analytical observations in support of intuitive judgment can and should be used as a calculation tool to trace and gain insights in the determination of which are the significant combat factors in order to identify relationships that might otherwise be obscured by the "fog of war."

BIOGRAPHIC SUMMARY

Professor Chantee Lewis is a graduate of the U.S. Naval Academy and has earned an M.A. in personnel administration from George Washington University, an M.S. in operations research/management from the Naval Postgraduate School, and a doctorate in business administration from George Washington University. As a naval aviator he has served as Commanding Officer of VAW-13; of Naval Air Station, Point Mugu, Calif.; and as a branch head in the Office of the Chief of Naval Operations. Professor Lewis is currently a member of the faculty in the Department of Management at the Naval War College.

NOTES

1. Answers - For 6 missiles, .06
10 missiles, .19
20 missiles, .65
30 missiles, .89

If events can be divided into two items of interest (hit, miss, or head or tails, et cetera) and if the series of trials are independent or near independent, then they can be represented by the binomial distribution. In this case the binomial expansion is of:

$\binom{N}{X} P^{(X)}(1-p)^{(n-x)}$ where N = No. of missiles
X = No. penetrating
P = Probability of a single penetration

2. If both are conservative, use similar value systems, and understand the opponents' choiees, then in order to avoid the worst possible outcomes the situation would be:

	A	B	C	D	E		Guarantee to you	
1	10	3	0	6	7		0	
2	3	5	6	8	10		3	
3	9	(6)	7	8	8	←	6	Max.
4	2	4	5	1	3		1	
Worst to Enemy	10	6	7	8	10			
		↑ min.						

3. The following are suggested for further reading:

Blackett, P.M.S. *Studies of War*. New York: Hill and Wang, 1962.

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Zehna, Peter W., ed. *Selected Methods and Models in Military Operations Research*. Washington: U.S. Govt. Print. Off., 1971. Excellent advanced text used by Naval Postgraduate School. Of particular interest, see chapter 6 relating to Lanchester's theory of combat.