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A Methodology for Measuring Support Outputs in Relation to Inputs-Productivity

Chantee Lewis
U.S. Navy

While finite funding necessarily demands greater efficiencies in defense management, the means to measure productivity in Government has long eluded the bureaucratic manager. This author has had experience in the development and application of a methodology that offers defense managers the opportunity to better utilize scarce resources.

A METHODOLOGY FOR MEASURING SUPPORT OUTPUTS IN RELATION TO INPUTS–PRODUCTIVITY

An article prepared

by

Captain Chantee Lewis, U.S. Navy

Statisticians have long maintained that there is no true way to measure productivity of government workers because their principal products–defense, administration of justice, furtherance of education, and other public goods are intangible and defy quantification. However, today's increased pressures for more efficiency in government–more public service for the dollar–are forcing economists to take another look at their assumptions. Although many say it still cannot be done, a great deal of research is being done in an attempt to utilize productivity analysis in government and defense the same way as we now apply it in private industry.

My remarks here will be primarily concerned with the measurement of productivity of shore station assets–with most of my illustrations taken from real-life observations aboard a large naval air station. Nevertheless, the principles and concepts which I will be discussing apply to any air station or shore station. First, we have a general question of what is productivity? Then, what causes change in productivity; how is it measured; and how can we do trade-offs between various asset alternatives? All of this should be of interest to managers and decisionmakers from both the private and public sectors.

To us in the military and to others as taxpayers, shore stations are big business! They are not as big in dollar values as the Navy's aircraft carriers; nonetheless, the annual operating cost of our naval air stations and naval stations is about $980 million; our annual investment cost for these stations (mostly in military construction money) is about $240 million; and our capital investment that is being actively utilized today represents over $10 billion. This is a sizable portion of not only Navy assets but the total defense assets. Now, with today's increasing cost of defense, the need for improved analytical aids to assist top management in making allocation decisions concerning the "best" mix of shore resources (in spare parts, support equipment, facilities, et cetera) is quite clear. It is quite possible that the labor costs (some 75 percent of the total budget) are rising faster than associated support and capital investment costs. It is likely that the "best" mix of resources to produce today's required shore station output is different than what, intuitively, would have been an effective allocation process a short time ago.

First, what is productivity? I regard productivity as a measure of management's efficiency or lack of efficiency in employing all of the necessary resources—natural, human, and financial. One could say productivity is the relation of outputs to inputs or, in a heavy labor intense situation, productivity is the producing of more with the same amount of human effort. It is more than the sweat of a man's labor! There is great room for an increase in productivity which depends upon many factors. The principal ones are: the factor of creative and innovative management; technology changes, which, of course, require changes in the policy on capital investment; and the worker's attitude which, to a great extent, is psychological. Productivity must be everybody's job. There are many values and payoffs for measuring and quantifying what I will call Federal, or defense, productivity. If you can define and evaluate the final specific units, i.e., specific units in relationship to the individual activity's goals or mission, then reorganize so that you can hold people accountable for the same specific areas and realize which areas are producing services which are not really final or necessary products and deemphasize these—then you have a good chance of improving your overall performance. If you are tracking or plotting the productivity through time series data, you can relate the impact of past actions concerning investment, the impact of changes in organization, span of control, and changes in, say, training programs to upgrade skills. Once you have surveyed the situation and reviewed the trends, then you can relate them to the individual management decisions and, in turn, influence future trends. Of course, productivity data can be useful to budget planning if you don't extrapolate the information too far. To translate your goals into specific activities requires that you identify your key or driving work centers that will require additional attention.

I use here as a point of departure some prior research done by Shallman, Sutton, and Lewis.[1] This research indicates that we have two general approaches for measuring government productivity. They are: (a) production functions of the Cobb-Douglas (C-D type) or the constant elasticity of substitution (CES type), and (b) indices of performance where routine tasks can be measured by the meaningful output-input ratios. However, considering the problems associated with obtaining real-life shore station cost data, one must look also at fixed and variable costs while confining one's efforts mainly to what we will call major cost centers.

The Cobb-Douglas production function permits one to establish relationships between the economic variables. Although this function has some drawbacks, it requires all inputs to be positively employed, and there is difficulty in handling calculations where the sum of the elasticities of substitution for all factors is other than one. Simplicity of the C-D production function and its neatness of fit with real-world data tend to bring many analysts to the conclusion that the C-D is preferred to the CES.

Now, let's expand and show to you just what the Cobb-Douglas production function is.

In figure 1 we have the classic Cobb-Douglas production function or a model for outputs in relation to inputs. It was devised almost 50 years ago. In this case the physical volume of output of goods and/or services is "P," the physical volume of input labor is "L," and the physical volume of capital is "C." In this case the capital is the most difficult item to measure. You must actually measure the capital in use, not the capital that is potentially available. The exponents "k" and "j" indicate the marginal productivities of the two broad factor input classes, and when time series data is used, we can obtain a statistical fit of the production function (using multiple regression analysis). The trend of scaler "b" indicates the rate change in productivity or, to look at it another way, the rate of change in productivity is the difference between the rate of change in output and in a weighted average of the inputs, using the exponents as weights. When the two exponents or elasticities are added together and if they equal one, we have what we call constant returns of scale. By this we mean if you increase your inputs by 10 percent, you may get an output increase of 10 percent, more than 10 percent, or less than 10 percent out, depending upon the returns of scale.

Going to real-life data and in the case of the agriculture industry, typical marginal productivities are as indicated. At the other end of the scale we have the electrical power industry which is heavily capital intense, and in this case the two exponents clearly sum to greater than one, indicating increasing returns to scale with the major factor being capital. It is interesting to note that the Cobb-Douglas type production function has been tested with data from many real-life situations, and, based on observations to date, it fits closely in most homogeneous production situations.

Having given you some feel for production functions and how they apply in the private sector, I will now swing into a practical application we have of this type of model involving a military maintenance problem.

This simultaneous equation model (figure 2) is an actual model that is running currently on the Center for Naval Analyses CDC 3800 computer. In this situation we do not have a single

$$P = bL^kC^j$$

P = Production
b = A Scaling Efficiency or Technology Change Factor
L = Labor
C = Capital
k = Elasticity with Respect to Labor
j = Elasticity with Respect to Capital

$k + j = 1$ Constant Returns to Scale
> 1 Increasing Returns to Scale

I.E. $P = 1.0 \; L^{0.75} \; C^{0.25}$ (Agriculture)

.
.
.

$L^{0.11} \; C^{0.97}$ (Electricity)

Fig. 1—Cobb-Douglas Production Function

output—we have a joint production function—with the assumption of independence to a degree between the two production functions for a reasonable range of inputs with a degree of substitution between inputs and outputs. In this case we have two specific outputs—U_1, sorties, and U_2, ready hours. We have a_0's as scaling efficiencies which are similar to our previous Cobb-Douglas model. For inputs we have W_1, W_2, W_3, and W_4, standing for aircraft, men, support equipment, and spares. The elasticities a_j's go with each W_j input. At this point an interesting computational problem starts. Do we have a relationship or coupling between each of the input factors ${}_1W_1$, ${}_2W_1$, et cetera? For example, I have observed that total labor goes jointly to produce the outputs U_1 and U_2 but not the specific amount of labor to produce U_2. In summary, I have observed "rim" values of these simultaneous equations. For each observation I observed a U_1, a U_2, a W_1, W_2, W_3, and W_4. The problem then is to *simultaneously* solve the above equations for the 18 unknown "cell" values, the a_0's, the ${}_1W_1$'s, the ${}_1a_2$'s, et cetera. A procedure to quantify the values for the cells in essence amounts to the following: (a) First, estimate any reasonable initial values for ${}_1W_1$, ${}_1W_2$, et cetera, such that they satisfy $W_1 = {}_1W_1 + {}_2W_1$ with all the W_j's being greater than 0. (b) Second, with the above initial values then obtain approximate estimates for the a_j's, using the likelihood estimation methodology.[2] In this case we are using the maximum likelihood estimates, recognizing that maximum likelihood estimators are not necessarily unbiased for small samples. (c) Third, the W_j's inputs are updated by a standard quadratic methodology. (Again see footnote 2.) (d) Fourth, update the a_j's by the likelihood estimated methodology, and (e) Continue this iterative process until a local minimum has been reached or approached. (f) Last, test for convergence or near convergence to a "best" local solution by varying the initial values over a wide range of starting values. In the case of real-world F4 aircraft data, the first local minimum solution is reached on the 12th iteration. After that, near similar locals can be reached around the 23d, 29th, and 35th iterations. The F4 data has been run up to a hundred iterations without a noticeable change in the pattern. In some cases we may have cycling occurring after the 25th iteration. Now let's look at some real solutions for the F4 (Figure 3).

$$U_1 = {}_1a_0 \; {}_1W_1^{\,{}_1a_1} \; {}_1W_2^{\,{}_1a_2} \; {}_1W_3^{\,{}_1a_3} \; {}_1W_4^{\,{}_1a_4} \; \epsilon_1$$

$$U_2 = {}_2a_0 \; {}_2W_1^{\,{}_2a_1} \; {}_2W_2^{\,{}_2a_2} \; {}_2W_3^{\,{}_2a_3} \; {}_2W_4^{\,{}_2a_4} \; \epsilon_2$$

$$\qquad\qquad \overline{W_1} \qquad \overline{W_2} \qquad \overline{W_3} \qquad \overline{W_4}$$

Where U_i = Outputs (U_1 = Sorties, U_2 = Ready Hours)

${}_ia_o$ = A Scaling Efficiency or Technology Change Factor

${}_iW_j$ = Inputs (Aircraft, Men, Support Equipment, Spare Parts)

${}_ia_j$ = Elasticity with Respect to the ${}_iW_j$ Input

ϵ_i = The Multiplicative Error Term

Fig. 2—Simultaneous Equation Model

The coefficient of elasticities for the a_i's and the percent of input resources W_i's are as indicated. Three samples of 30 observations each in all were drawn from the total observations, each with a different average level of ready hours. For these samplings the coefficient of determination (R^2) ranged from .884 to .926. The residuals were analyzed for three types of possible error. No sampling bias could be detected. The variance proportion of the residual error was about .263 and the covariance about .737. A plot of residuals indicated some difficulty with outlier observations, particularly from those of squadrons aboard the U.S.S. *Kennedy.*

Having established the marginal productivities of these four inputs and the ratio of their inputs, let us look at a way that we can use this information.

The breakdown of output per input in relation to price is as indicated in figure 4. We see that with an increase of all inputs by 10 percent the support equipment (that was reported in the 3M data) accounts for an increase of 6.886 sorties which requires 8.7 more units of support equipment at a cost of .666 thousand dollars per unit per month, or the marginal output is 1.188 sorties per K dollar spent on support equipment. In a similar fashion we have compared the marginal productivity of spares, aircraft, and men and divided them by this price and ranked the marginal physical products to cost. Conclusion, on the margin invest in support equipment first and total manpower last for the F4J.

Having established the marginal productivity of the various inputs, I have also established a Cobb-Douglas objec-

	Aircraft (W_1)		Men (W_2)		Support (W_3)		Spare Parts (W_4)	
	a_1	W_1	a_2	W_2	a_3	W_3	a_4	W_4
Sorties	.044	.25	.033	.63	.025	.57	.025	.23
Ready Hours	1.090	.75	--	.37	.420	.43	.120	.77

R^2 was .884 ⟶ .926

Analysis of Residuals - U^M ~ .0, Bias Proportion
U^S = .263, Variance Proportion
U^C = .737, Covariance Proportion

Fig. 3—Coefficients of Elasticities ($i^{a}j$'s) and Percent of Resources ($i^{W}j$'s) for the F4J Aircraft

$$W_3 \quad \text{(Support)} = \frac{6.886}{(8.7)\quad(.666)} = 1.188 \text{ Sorties/K}$$

$$W_4 \quad \text{(Spares)} = \frac{2.454}{(7.9)\quad(8.69)} = .357 \text{ Sorties/K}$$

$$W_1 \quad \text{(Aircraft)} = \frac{17.319}{(1.2)\quad(46.2)} = .312 \text{ Sorties/K}$$

$$W_2 \quad \text{(Men)} = \frac{0.894}{(14.5)\quad(.466)} = .132 \text{ Sorties/K}$$

Fig. 4—Ratio of Marginal Physical Products to Cost: F4J

tive function and maximized it under various constraints—such as a constant budget. In the case of the F4 we have been able to get an increase in productivity of 6.8 percent (18.39 sorties per month) over the base case.

Now let's look at the *other general approach* which I will call indices of performances. The use of productivity ratios or indices is somewhat simpler than production functions. It involves estimating productivity as a ratio of output to a weighted average of an associated input.

Again you have to be careful in the measurement of capital charges; in some cases they are reported as gross, others as net depreciated charges. A key point is that outputs and inputs must be measured consistently.

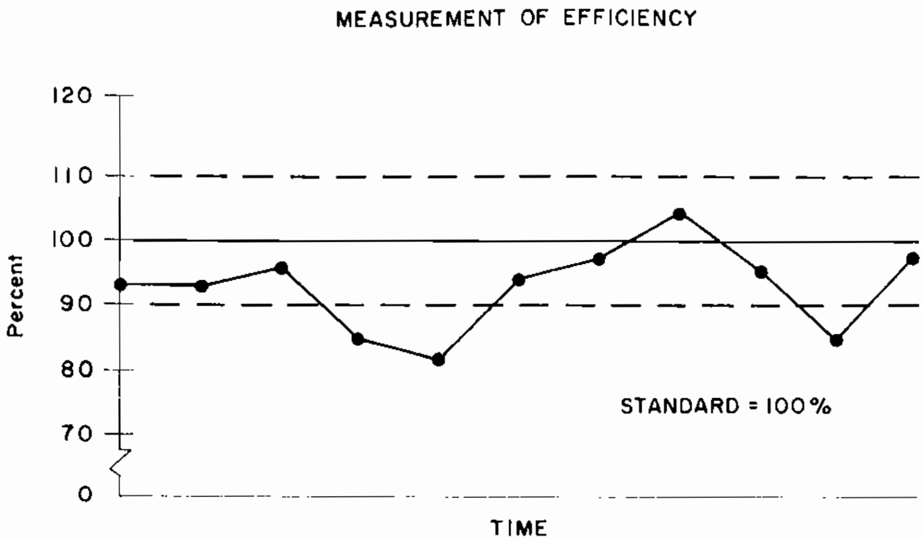
Now how do we get started in working up the productivity indices? First you must remember that you don't want to force the data, otherwise you're liable to get bad observations or just "gun-decked" information—this means that individual activities should be asked to submit only readily available data and not make the statistical computations in the field.

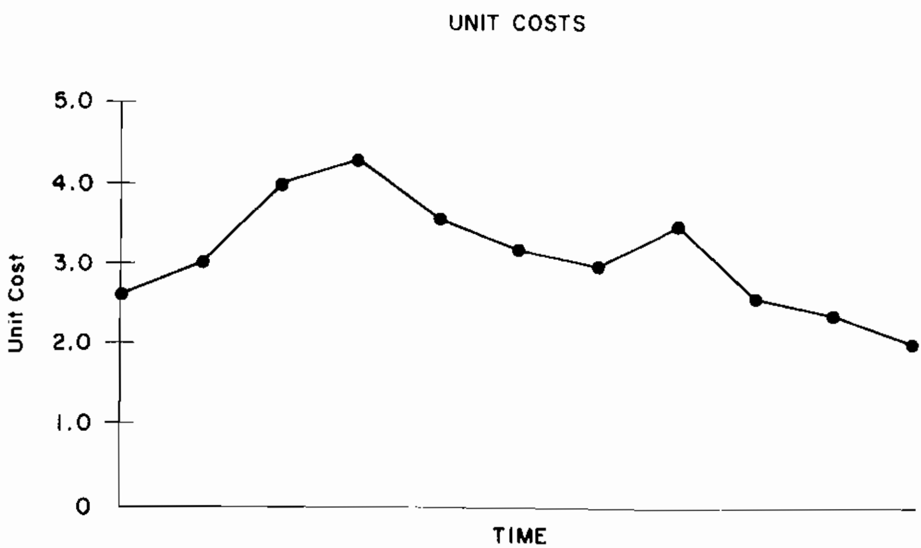
Figure 5 shows an overall productivity of a major function such as reported to top management. The ranges (acceptable tolerances shown by the dotted lines) provide a basis for evaluation of the function's performance as warranted—usually on the management by exception basis. The bottom chart depicts unit costs—the tracking of unit costs is an essential aspect of management control. This index can be one of the most useful cues to alert us when things are changing and deserves special review.

Figure 6 displays, say, a year's production by work units over time. The solid line depicts the actual workload, while the dotted line is a 3-month moving average which smooths out the peaks and valleys and produces a better basis for trend analysis or forecasting in the short term. I have found from my actual observations that wherever the actual workload line crosses in a *downward trend* the 3-month moving average, it is an advance cue that you may be starting to lose control. You probably want to immediately expand your search for additional information and determine if this is due to the perversity of the reporting system or a result of external shifts in resources, policy, personnel, et cetera. The bottom chart shows the productive personnel equivalents used to accomplish the workload displayed. If the activity is 100 percent effective, the dotted line, which represents the standard equivalents, must be identical with the solid line, which represents actual equivalents. Now, I am going to leave for a moment these general discussions of indices of performance and look at a comparison of fixed and variable costs.

You will recall from the earlier figure on production functions, one of the things that I was interested in was the summation of the elasticities of all inputs and if they added up to greater than one I had increasing economies of scale, and if they added up to less than one I had decreasing economies of scale.

In figure 7 we have plotted several real-world air stations and on the x axis we have the ouput which is tenant demands based on a weighted average of the tenant population (people) and square footage of shop or hangar space. On the y axis we have the cost to accomplish these outputs, and since the stations vary considerably in magnitude of actual budgets, we have the variable budget as a percentage of the total budget. Now, we haven't actually observed the stations operating over the full range of output. But for the general range of varying workload that has been observed, the portion of the cost that is fixed and what portion is variable is as indicated. In the case of NAS Norfolk, about 43.8 percent of its cost is fixed for the range of data that we have





Unit cost may be the most useful decisionmaking tool

Figure 5

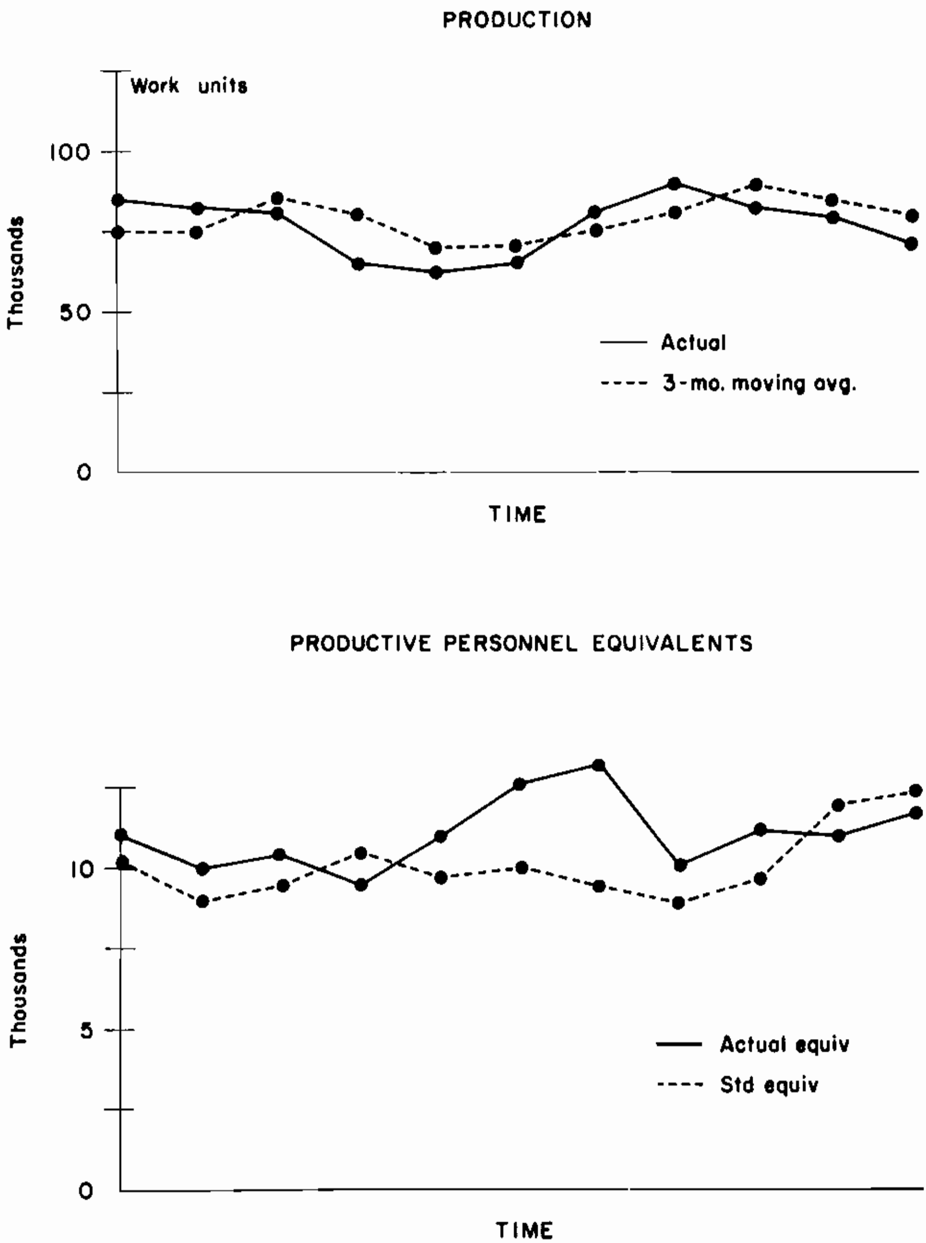

PRODUCTION
Work units
100
50
0
Thousands
TIME
Actual
3-mo. moving avg.
PRODUCTIVE PERSONNEL EQUIVALENTS
10
5
0
Thousands
TIME
Actual equiv
Std equiv

Figure 6

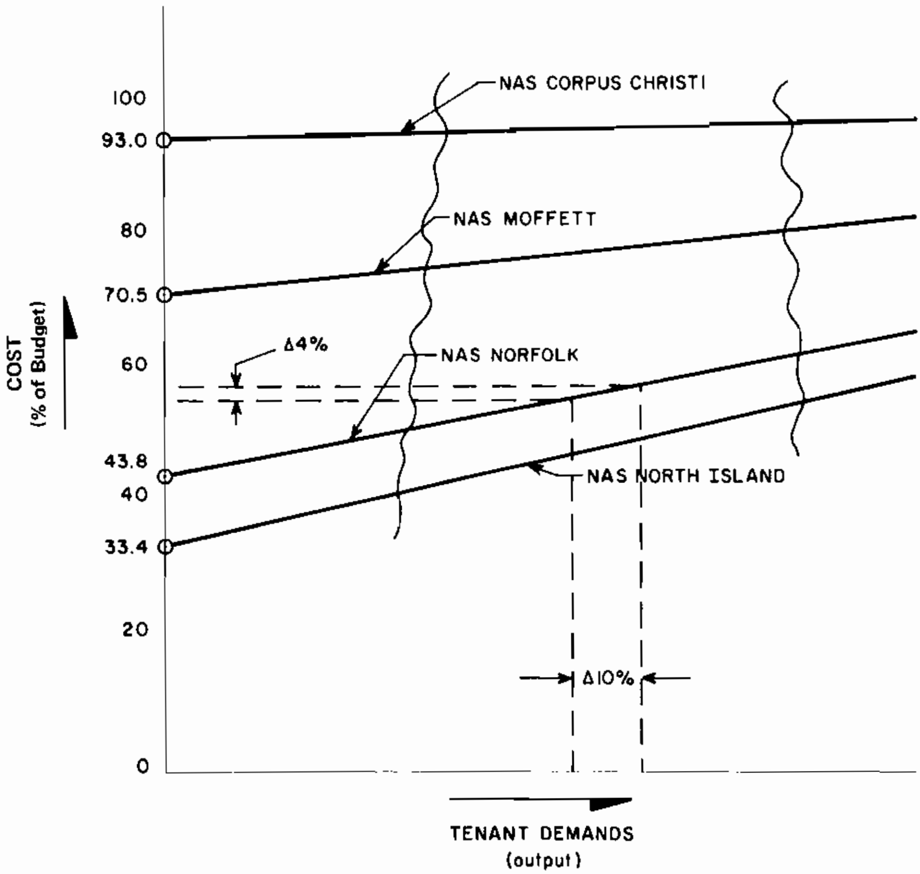


Fig. 7—Fixed and Variable Cost of Naval Air Stations

Source: INS - 0340 - 72 of 13 March 1972

recorded. If Norfolk had a 10 percent increase in demands from the tenants, the inputs for the station would only need to be increased by about 4 percent. This does indeed indicate that we have economies of scale at some large air stations. You will notice that NAS Moffett and NAS Corpus Christi appear to be mostly fixed costs, and their budget needs are somewhat indifferent in output demands. This figure not only indicates where we have increasing returns to scale, but also it appears, if our data is reasonably reported, that an air station should usually be operated in one of two ways—with a very heavy workload or not at all (deactivated) if economic efficiency is the major criterion. With this cue as to how our elasticities of inputs may sum up—let's look for where the big dollar items are.

Figure 8 shows a ranking, by departments, of the NAS Mugu elements of the annual budget less MILCON. It does not include military labor, which is about another 20 percent. You will notice the first three departments (Public Works, Supply, and Air Operations) and then Surface Craft (which in the case of Point Mugu does similar functions to Air Operations Department) together add up to about 85 percent of the total budget. Within these four departments I will make trade-offs be-

	Labor	Overtime & Travel	Material	Total	NAS % of Budget	Culminate
Public Works	$ 6,448	$104	$2,905	$ 9,457	54.3	54.3
Supply	2,644	76	260	2,980	17.1	71.4
Air Operations	1,696	17	154	1,867	10.7	82.1
Administration	790	32	130	952	5.5	87.6
Security	785	26	20	831	4.8	92.4
Surface Craft	275	7	160	442	2.5	94.9
Communications	360	2	24	386	2.2	97.1
Medical/Dental	260	0	100	360	2.1	99.2
San Nicolas Island	122	1	12	135	0.8	100.0
Totals	$13,380	$265	$3,765	$17,410		
	76.8	1.6	21.6			

Fig. 8—Ranking of Departments by Size of Annual Budget (Less Mil Con)

		Output	Units	Inputs
Operations	U_1 -	Special Projects (I.E. Target Pickup)	A/C Helo Hours of Service in Relation Scheduled Operations	Aircraft, Space Parts Support, Maintenance Manpower
	U_2 -	Runway Use	Landing (Down Time and Delays)	GCA, SAR, Crash Crew Towers Ops, Maintenance
	U_3 -	Transit Service	Passengers/Crew and Tons of Cargo, Weighed for Delays	Aircraft Equipment, Manpower

Fig. 9—Measures of Merit

tween input choices or alternatives and for the various constraints maximize the outputs. It is interesting to note that the total labor cost is 76.8 percent. We have a very labor intense situation and, regrettably, considering the way wage rates are changing, relative to material cost, we are becoming more labor intense. When comparing like departments in the commercial airport management area, the Navy appears more labor intense than the commercial sector. This has also been confirmed by separate research done by the Air Force which found defense airfields less capital intense than airports in the private sector. Since the private sector may be more sensitive to relative price changes, this makes me suspicious, and we may be looking upon our labor somewhat as a "free goods," or we probably are overinvested in labor relative to putting the next dollar in the capital area.

Now with this idea of what the typical large departments might be and how total elasticities of economies appear to be (ϵaj's > 1), let's look at one of these departments in detail.

Figure 9 shows the three simultaneous outputs for the Air Operations Department--10.7 percent of our total budget. The output units and the input units will be amplified in the next figure.

In figure 10 we have the output index of Special Projects or helicopter flights by the months actually on the range, launching weapons, and recovering targets. A successful mission counts as one unit of output—this places the incentive on management to use the helicopters effectively in order to minimize delay times, standby times, transient times, et cetera, as all of these other items are intermediate outputs. You will note that we have an upper and lower threshold. If we exceed this upper limit, it is a cue to management that possibly we are not meeting the NATOPS standards, may be cutting corners in standard maintenance, or that we may be underutilizing the surface

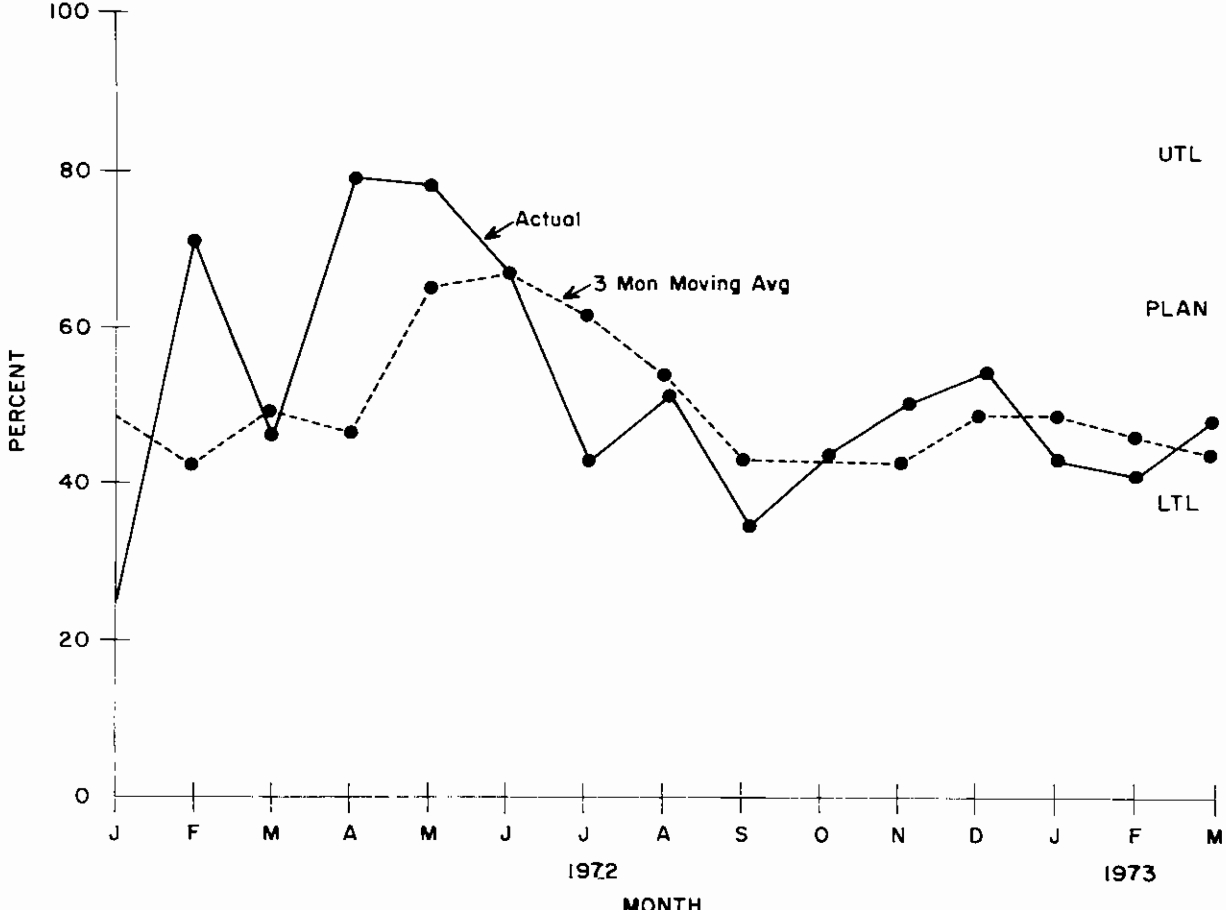


Fig. 10—Special Project Flts./Mon

craft. A trade-off exists between helicopters and surface craft at Mugu. The lower threshold is based on the average expected limited availability of aircraft and pilots. Should we fall below this, there is strong indication that there is need for further investigation. We may have too few demands on our helicopters—maybe we should turn in some of these assets for other alternatives within the Navy. We may be overworking the boats, or external factors (such as support equipment, station supplies) may be driving the problem. From a management standpoint, however, the most interesting index is the 3-month moving average as indicated by the dotted line. The moving average is very important because, on a short-term basis, external factors are always changing the expected long-term trends. As long as the actual observations are above the moving average, I would monitor this department by "management by exception" rule. As soon as the two lines cross, such as shown in figure 10, in March and July—the downward direction is an immediate cue to review and analyze *in detail* why the output has deteriorated from a statistical viewpoint. It may be due to just the perversities of our recording systems. But so far, in every case when the lines did cross in a downward direction, it was due to adverse changes in supervisory and maintenance personnel, changes in customer service, changes in handling of spare parts, changes in specific support equipment, or just changes in the scheduling procedures. All of this requires management's quick attention to get the situation turned around. In summary, this chart is an index to encourage the Operations Department to be productive and establish threshold limits that should not be exceeded. As we try to properly utilize our resources and at the same time watch the 3-month moving average, we have a good index to indicate progress and to alert us when the possibility exists that we are standing in difficulty, before the situation is actually out of control. Then with this alert time we should be able to control, to a degree, the direction of our inputs or make necessary changes of our policies. In a similar fashion each of the other three departments' outputs were plotted through time, and, where possible, charts were made of their unit cost of outputs in order to monitor and direct the individual departments. The measuring and monitoring of productivity in relation to the final objectives is most important. Research by others indicates that a great many of the factors that affect the activity are controllable by management.

While the planning and controlling of productivity is desirable, one should be aware of certain cautions when using production indices. The workers and supervisors often misunderstand their intended use, and the performance factors can appear to those in the field as being too technical or mysterious. We must be aware that prior productivity measurement programs have "forced" reductions in manpower, in spite of increasing workloads, without identifying the need for corresponding labor-saving investments. In the long run, these prior efforts have led to a degradation in the quality of output.

Employees are suspicious that once the indices have been established, top management may "freeze" the program and exclude new information that might warrant later changes in the methodology. It appears that at times in the past the indices were tied primarily to costs without any cross-reference to the associated benefits.

Evaluation of the accuracy and the validity of the data is essential if the system is to effectively provide meaningful and useful information. In addition, independent reviews should be made from time to time to satisfy management that the standards originally established are still valid.

Again, remember that in any government measuring system we have the complication of the disincentives which are built into the system by law, custom, or tradition.

With these limitations in mind, it appears that productivity is a task for all—everyone must participate in the establishment of the units of output that relate to their *end objectives.* From this we must establish our initial hypothesis for the production function model that is to be tested—then collect the data and test. Control limits must be established over this model, specific managerial responsibilities *must be fixed* and then monitored with the necessary redirection of assets.

Since the outputs of most naval activities are too heterogeneous for normal production function measuring techniques, partial indexes or productivity indexes probably are the only practical measure. When using indexes, remember that an increase in output per worker man-hour may be due to many factors. Under no circumstances can the change in the productivity index be assumed to reflect changes only in the efficiency of the primary factor itself (labor). You have to consider the simultaneous role of the other factor inputs, then if you have established a reasonable model—(recalling technology, management/union policies, and work procedures are always changing) —yesterday's hypothesis probably will not fit today's needs. You must update your model from time to time.

Productivity measurement has proven itself in the private sector—all indications are that it is a useful management tool in the public sector. The potential application of this methodology is manyfold. It is a method to increase potential output for a specific budget—a way to expose underutilized resources—or a device to increase management control/motivation of the *total* work force. Remember, as in the rest of society, resources in government are scarce! We must strive for increases in productivity.

BIOGRAPHIC SUMMARY

Capt. Chantee Lewis, U.S. Navy, is a graduate of the U.S. Naval Academy and has earned an M.A. in personnel administration from George Washington University, an M.S. in operations research/management from the Naval Postgraduate School, and a doctorate in business administration from George Washington University. As a naval aviator, he has served as Commanding Officer of VAW-13; of Naval Air Station, Point Mugu, Calif.; and as a branch head in the Office of the Chief of Naval Operations. Captain Lewis is currently chairing the Tactics Department of the Naval War College.

NOTES

1. William S. Shallman, U.S. Army Management Engineering Office, Rock Island. His main work appears in U.S. Congress, Joint Economic Committee, *Measuring and Enhancing Productivity in the Federal Sector* (Washington: U.S. Govt. Print. Off., 4 August 1972); F. Scott Sutton's research appears in "Aircraft Pipeline Study: An Applied Model for Determination of Minimum Cost of Aircraft Pipeline Factors," both for (Arlington, Va.: Center for Naval Analyses, June 1969 and May 1970). Chantee Lewis, "The Use of Simultaneous Equation Models for Decisions Pertaining to the 'Best' Mix Between Aircraft, Spare Parts, Support Equipment, and Support Personnel," Research Contribution 206 (Arlington, Va.: Center for Naval Analysis, May 1972).
2. Lewis, pp. 35-40.