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Game Theory

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GAME THEORY

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by

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A. *Introduction*

Current planning for the employment of forces in being has to proceed in the face of incomplete intelligence about the numerical strength of the opposition. Estimates of the enemy's operational unit performances tend to be mirror images of our own, which are all too often poorly understood in the present era of rapid weapon development.

Planning for the future has to contend with these same uncertainties in an aggravated manner. It is further handicapped by the unpredictability of technological trends and of the political context five or ten years hence. These indeterminacies make the important procurement and R&D decisions particularly difficult ones.

In addition, statistical fluctuations affect the outcome of any engagement since chance variation is always with us.

Finally, there is the ever-present question of enemy intentions. How will he elect to employ his capabilities to counter our courses of actions? Clearly, enemy reaction has to be considered in all military planning.

This week you are studying one approach to this vexing problem, War Gaming. I am going to introduce you to another one this morning, Game Theory.

Here is a quick run-down on what I am going to cover. I am going to tell you briefly about the origin of Game Theory, define a few terms and develop the basic concepts of the theory by way of an example. I will then show how game theoretical notions are actually being used today in some areas of Naval planning. Finally, I shall make some general comments upon the implications of the use of Game Theory in the planning process.

Since Game Theory is a branch of mathematics I will have to use quite a few charts, graphs and figures. But I promise: No integral signs.

B. The Origin of Game Theory

Game Theory was invented by the late John von Neumann, a very versatile scientist who contributed to H-bomb development and served as an AEC commissioner.

Von Neumann became interested in the analysis of conflict situations a little more than thirty years ago when he was still a student in Budapest. Real life conflicts were at first quite intractable. Von Neumann therefore used a common scientific dodge and studied a more manageable model instead, a stripped version of poker—not strip poker though.

Von Neumann's poker game was reduced to bare essentials to facilitate analysis. For example, only two players are involved. Still the game has enough similarity to more serious conflict situations to be a useful model. Both players try to win at the expense of the other. Neither controls the game by himself. For instance your opponent may not put any money into the pot when you have a good hand and he has a poor one. Decisions have to be made on the basis of fragmentary information. You have to bet without looking at the other player's hand. Finally, there is a statistical element in the game, the order of cards in the deck.

Because of the chance factor probability theory is needed in poker as well as in most military conflict situations. But probability alone is not enough since it does not really help in outguessing the other fellow.

Von Neumann's model studies of poker gave him new insight into the fundamental nature of conflicts and led to the formulation of Game Theory. Game Theory is more ambitious than probability theory. It tries to provide a rational basis for action in the face of intelligent opposition.

Game Theory is not the first instance where a new field of mathematics began with a study of parlor games. The same was true of classical probability theory whose potentialities for insurance underwriting were recognized only after a couple of hundred years.

The accident of birth explains the frivolous name of the theory. It also accounts for some of the terminology. Opponents are players. Players may be individual combatant units, fleets, or whole nations, depending upon the nature of the problem. The only requirement is a common goal. The rules of engagement are the game. A single contest is a play of the game. The outcome determines the payoff. The terminology also has military overtones. The players' courses of actions are strategies, or occasionally tactics.

C. *Game Theoretical Notions*

Let us see next how Game Theory can help in a typical military problem: The determination of an aircraft configuration. The numbers in the example are hypothetical but the problem is a perfectly real one. Here is the situation:

BLUE can equip his bombers with guns or ECM and use high altitude profiles. He can also strip his aircraft in order to penetrate on the deck. The defense, RED, also has several options: A standard interceptor, a low-altitude version, or a fighter with ECCM.

The problem then is a dual one: The best choice of bomber and fighter configurations considering the opposition's range of options. The bomber command wants to maximize his penetration probability while the fighter command wants to minimize this same quantity. Because of lead-time factors both sides have to make their system choices at the same time and in ignorance of that of the opposition. To analyze this situation let us write the nine possible outcomes in the form of a pay-off matrix (Fig. 1). On the left are the three alternatives of the offense. Across the top are the defensive choices. The nine numbers in the matrix are the penetration probabilities. For example, an ECM equipped bomber has a 50% chance of getting through when the defense uses a standard fighter.

By the way, I am going to use the same color convention for the rest of the morning. The maximizing player is always BLUE. His options are listed on the left of the matrix. The minimizing player is RED. His choices are across the top.

Being conservative, the offense looks at the worst that can happen to them, the row minima marked by dots. The largest of these minima in the middle row is called the maxmin and equals 70%. It determines BLUE's safest option. If BLUE settles on stripped bombers, he assures himself of getting through at least 70% of the time.

Conversely, the defense looks at the worst from their point of view, the column maxima, marked by stars. The smallest column maximum in the middle column is called the minmax. It determines RED's safest choice. If RED has low-altitude fighters, he can hold BLUE's penetration probability to 70% or less.

RED FIGHTER

BLUE BOMBER:	ST'D	LOW ALT	ECCM
GUNS	• 40	50	60
STRIPPED	80 ★	•• 70 ★★	90 ★
ECM	50	60	• 40

• ROW MINIMA ★ COLUMN MAXIMA

•• MAXMIN = ★★ MINMAX

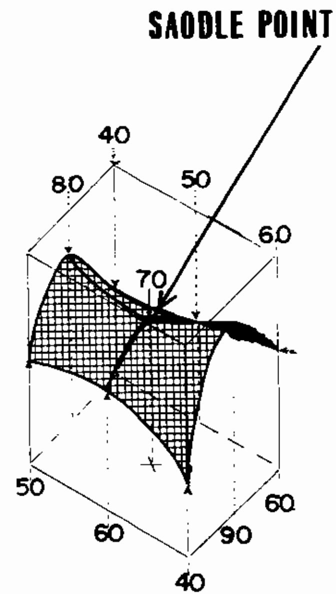

FIG. 1

BLUE BOMBER \ RED FIGHTER	ST'D	LOW ALT	ECCM
GUNS	40	50	60
STRIPPED	70*	••50	80*
ECM	50	60**	•40

NO SADDLE POINT

• ROW MINIMA   * COLUMN MAXIMA

•• MAXMIN ≠ ** MINMAX

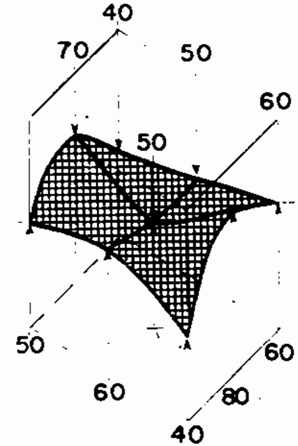

Figure 1 says, in effect, that BLUE should buy stripped bombers and that RED needs low-altitude interceptors. The penetration probability is 70%. This solution is safest for both sides. Bombers with guns or ECM could do worse for BLUE if RED has this low-altitude fighter. Similarly, high-altitude fighters would not do well against the stripped bomber.

Notice that the maxmin equals the minmax in this matrix. A matrix for which this is true is said to have a saddle-point. The reason for this terminology is shown in the little diagram on the bottom of Figure 1. Pay-offs are represented by vertical spikes, with a surface put over the tops. The surface looks like a saddle. The saddle-point is in the center where the penetration probability is 70%.

Whenever the maxmin and minmax are equal there is a nice stable situation. Both sides try to get on the saddle-point. This is equivalent to conventional capabilities planning. There is really no temptation to do anything else.

Saddle-points are not very common, especially in large matrices with many rows and columns. Figure 2 is an example of a matrix without a saddle-point. It was derived from Figure 1 by improving the RED's ground environment and thereby making low-level penetration less effective. Again, BLUE marks the row minima with dots and RED the column maxima with stars. The maxmin and minmax are now different. The pay-off surface given on the lower right of Figure 2 no longer looks like a saddle. This makes the problem much more complicated.

BLUE might figure that he can do better by buying ECM bombers rather than stripped bombers which are really his safest choice. But RED can punish him for doing this by adopting ECCM fighters, if he correctly guesses BLUE's intentions. In other words, there is a conflict between planning on capabilities or intentions.

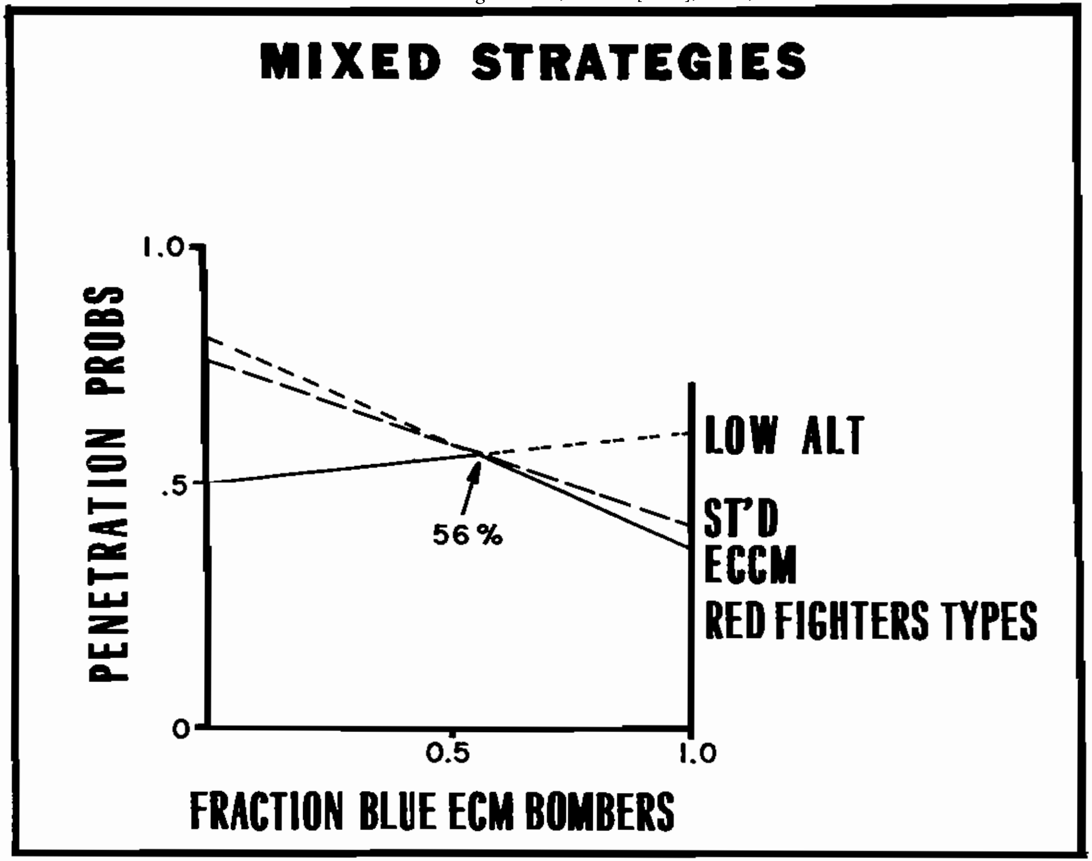
MIXED STRATEGIES
PENETRATION PROBS
1.0
.5
0
0.5
1.0
56 %
LOW ALT
ST'D
ECCM
RED FIGHTERS TYPES
FRACTION BLUE ECM BOMBERS

FIG. 3

Game Theory provides a rational way out of this dilemma. First we note that the pay-offs in the top row are never larger than those in the middle row. BLUE has nothing to gain from bombers with guns, regardless of RED's choice of fighters. We can forget about the top row. But we cannot eliminate any other rows or columns in this simple manner.

Let us now see what happens if BLUE uses stripped and ECM bombers in varying proportions against each of the three fighter types in turn. The analysis is shown in Figure 3. The horizontal axis gives the fraction of ECM aircraft in BLUE's mix. The three lines give penetration probabilities against RED low-altitude fighters, standard fighters and ECCM fighters, respectively.

The intersection is best from BLUE's point of view. It assures him of a penetration probability of 56% with a mixture of 60% ECM aircraft and 40% stripped bombers. A higher fraction of ECM bombers could lower BLUE's penetration probability if RED had ECCM bombers. A lower fraction would reduce the penetration probability in case RED had low-altitude fighters. A similar construction from RED's point of view shows that he should buy 20% ECCM bombers and 80% low-altitude bombers. This mix will in turn insure for RED that the penetration probability does not go above 56%. These mixes again equalize the maxmin and the minmax and stabilize the situation. This analysis assumes that RED cannot match his fighters to BLUE's bomber at intercept time.

The mixed strategies are improvements for both sides over their respective safe pure strategies. BLUE raises his assured penetration probability from 50 to 56%, while RED reduces his maximum risk from 60 to 56%.

In this particular case gains from using mixed strategies were not spectacular. But, of course, every

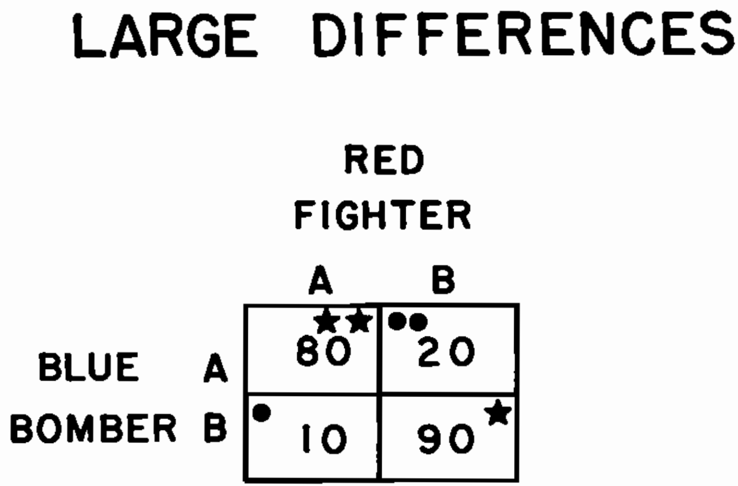

FIG. 4

little bit counts. On the other hand, the gains can be very large when the conditions are right. Figure 4 is a matrix where this is so. The maxmin and the minmax differ by 60%. The penetration probability with the appropriate mixed tactics is 50%. It represents improvements of 30% for each side.

That mixed strategies are, in fact, best for both sides when there is no saddle-point, is one of the central results of Game Theory. Mixed strategies are, of course, only appropriate when both sides have to make decisions in ignorance of those of the opposition. Now the concept of mixed strategy is really not entirely new—you use it intuitively in poker when you consider to fold, call or raise with a poor hand. Game Theory can prove that your intuition is right. You have to bluff some of the time in order to win. But it has not gotten very far in telling you how often to bluff with a given hand. The game has too many ramifications for detailed analysis. The situation in military problems is similar. Some of the simpler situations like the one we have just talked about can be analyzed explicitly. A few solutions have become an integral part of tactical doctrine. I am going to give you an example in a moment. Von Neumann himself had a hand in planning Operation STARVATION, the eminently successful mining campaign against Japan. We get into trouble in case of broader strategic problems. The situations are just too complex. Still Game Theory can often provide a framework for qualitative rather than quantitative study of the situation. This alone is worthwhile. It helps to visualize the consequences of various enemy actions. Clarifying the merits of mixed strategies has been a real achievement of Game Theory. I believe that the potentialities of mixed strategies for combat situations were hardly recognized twenty years ago.

D. A Tactical Application

Let us now see how game theoretical notions have been applied in the formulation of Navy tactical doctrine. The example is one with which you are all familiar, Antisubmarine Screening. But you may not have known that Game Theory played a part in the solution of this problem.

Figure 5 illustrates the general situation. The surface force has to guard against two forms of submarine attack, torpedo spreads launched at long range from outside the screen and salvos fired from near by after the submarine has successfully penetrated the screen. With ample forces, the screen clearly belongs on the boundary of the torpedo danger zone. But with insufficient forces a screen that far out would be practically useless. Escorts have to be brought in closer. When they are too close submarines can take a free shot from outside. The best screen position is somewhere in between. It should minimize the submarine's probability of success. We can find it graphically, as shown in Figure 6.

We plot the submarine's success probability for the inside attack for various screen positions and superimpose the probability of success of the outside attack, delivered from just beyond the zone of sonar coverage. This kind of graph is called a decision diagram.

The solid curves give the damage probability as a function of screen position assuming that the submarine uses his better mode of attack. The lowest submarine success probability is at the intersection of the two probability curves. The crossing is a minmax. It determines the safest defensive disposition. It doesn't matter to the surface force how the submarine attacks when the formation is screened in accordance with this construction. If the screen is farther out submarines should always penetrate. If it

ASW SCREENS

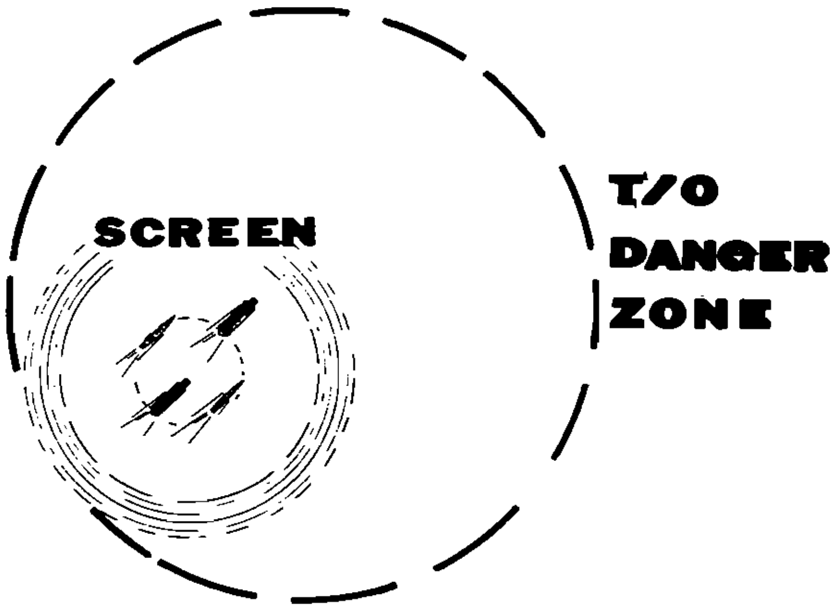

FIG. 5

DECISION DIAGRAM

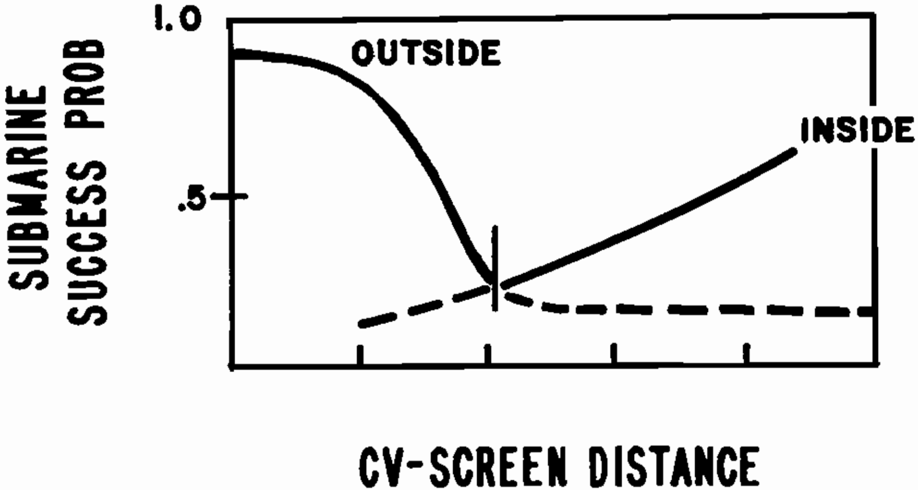

FIG. 6

is moved in they should fire from outside. Smart submarines can inflict unnecessary losses when the surface force departs from its safe screen disposition in either direction.

The screening problem is a special case. The submarine can see the defensive disposition before committing himself and determine on the spot whether to penetrate the screen or fire from outside. The surface force still has to watch how submarines conduct their attacks. If submarines rarely penetrate, the screen should be moved out to meet them. On the other hand, if submarines tend to penetrate frequently, the screen should be contracted. The surface force can get an advantage in either case. Submarines have to guard against this contingency by also mixing their tactics and using inside and outside attacks about equally often, when the screen is placed correctly. Game theoretical solutions are really just points of departure for the beginning of a war. They may need adjustment after the enemy discloses his doctrine in actual operations.

E. A Strategic Problem

So much for tactical applications. In the next few minutes we will explore some of the nasty difficulties we can get into in broader military problems. To do so I am going to acquaint you with Colonel Blotto and his Dilemma. Colonel Blotto is a mythical character invented by 19th century war college strategists. He was later adopted by the game theoreticians. Colonel Blotto appears, for example, in a very nice "primer" of Game Theory by Williams, called *The Compleat Strategyst.* Several copies are in the library. You may be interested to look at one this week.

The Blotto Dilemma is a simple model of the basic strategic problem, the disposition of forces. Actually our example should be called Admiral Blotto's

Dilemma, since it is placed in a Naval setting. Admiral Blotto is a BLUE cruiser commander. We will analyze his problem by following the steps of the standard "Estimate of the Situation":

1. Mission

2. Situation and Courses of Action

3. Analysis of Opposing Courses of Action

4. Comparison of Own Courses of Action

5. Decision

Blotto's *Mission* is to attack RED shipping. Here is the *Situation*. BLUE has two cruisers. Right now two RED convoys are at sea, some distance apart. They are covered by a total of three cruisers. BLUE does not know where the RED cruisers are. BLUE has two *Courses of Action*. He can either send both cruisers out together to attack one of the convoys or he can attack both with one cruiser each. RED in turn is aware of the nature of the threat. He also has two *Courses of Action:* To cover one of the convoys with three cruisers and leave the other one unprotected or give one convoy an escort of two cruisers and the other one an escort of one. Both commanders really have the same basic dilemma: To concentrate their forces or to divide them.

To reduce the problem to the bare essentials we make some sweeping assumptions. The first two are: The two convoys are equally important and all cruisers have the same fighting power. Others will be brought in later.

Writing the *Opposing Courses of Action* in matrix form greatly facilitates the analysis (Figure 7). Again, BLUE's courses of action are on the left, RED's across the top. The entries in the four boxes list the

Analysis of Opposing Courses of Action

BLUE Options	RED Options: Concentrate	RED Options: Divide
Concentrate	2 BLUE CL meet either a convoy protected by 3 RED CL or an unprotected RED convoy	2 BLUE CL meet either a convoy protected by 2 RED CL or a convoy protected by 1 RED CL
Divide	1 BLUE CL meets a convoy protected by 3 RED CL The other meets an unprotected RED convoy	1 BLUE CL meets a convoy protected by 2 RED CL The other meets a convoy protected by 1 RED CL

Figure 7

Pay-off Matrix

RED Options

BLUE Options	Concentrate	Divide
Concentrate	$D_2 - 2C$	$\frac{3}{2} D_2 + 1C$
Divide	$2D_1 - 2C$	$D_1 - 2C$

(C, D, and D_2 are defined in text)

Figure 8

possible encounters. If both commanders concentrate their forces, BLUE's units will either find an unprotected convoy or a strongly guarded one. Chance will govern which one of these two events will occur. If both sides divide their forces, one of the BLUE cruisers will encounter two RED cruisers and the other just one. It should be clear how the other two boxes are filled in.

Unfortunately, the matrix of encounters does not really help us. We have to go further and estimate the pay-offs for each box. To do this, we need some further assumptions. The raiders have to fight the escorts before they can attack the convoys. When equal forces meet either side has a 50-50 chance of winning. The most likely outcome of an encounter between unequal forces is that the stronger side will win without losses. To simplify the matter, let us ignore the less likely outcomes. Let us also say that BLUE inflicts damage D_1, when one cruiser gets through to a convoy. Two cruisers can inflict damage D_2. All cruisers are valued equally, at C. Sinking a RED cruiser adds to BLUE's pay-off; losing one detracts from it. With these rules, we can compute the pay-off matrix from BLUE's point of view.

The algebra is straightforward but a little messy. I will not take time to go through the calculations now. The results are listed in matrix form in Figure 8. It becomes the basis for BLUE's *Comparison of Own Courses of Action.*

BLUE wants to maximize the pay-off, regardless of RED's course of action. He has to assign relative values to D_1, D_2 and C before he can do this. You remember that C is the value of a cruiser. D_1 is the damage inflicted on a convoy by one cruiser. D_2 is the damage inflicted by two. As you will see in a moment, assignment of values is really crucial, since it strongly affects the *Decision.*

The relation between D_1 and D_2 depends upon the rules of engagement between cruisers and convoys. We already dealt with the same kind of problem a moment ago, when we made assumptions about the outcomes of surface actions between cruisers. If a single cruiser can destroy the convoy after it has overcome the covering force, one cruiser can cause just as much damage as two. D_1 and D_2 are equal. This would be true, for example, in case of a small troop convoy. On the other hand, a large mercantile convoy would probably scatter over a wide area before it could be brought under attack. Two cruisers would then be able to sink twice as many ships as one. D_2 would be $2.D_1$.

Determining the relation between the D's and C is even harder. Simple mechanical rules that equate the loss of a cruiser to the destruction of some fixed number of merchant ships in a mechanical manner are wholly inadequate for this purpose. Fundamental considerations of military worth are involved. For instance, place yourselves in the position of the German admiral at Brest, who had to weigh the potential gains from sending SCHARNHORST and GNEISENAU out on a raid against the risk of losing the fleet-in-being. Still, problems of relative military worth have to be faced before coming to a rational decision. This is true, though, whether you use Game Theory or not.

Once the relationship between the D's and C is settled, resolution of the dilemma becomes straightforward. This is shown by the three matrices in Figures 9a, 9b, and 9c. They are derived from the original pay-off matrix in Figure 8 by substituting the assumptions about the D's and C, that I just discussed. The maxmins are again marked by two dots, the minmaxes by two stars. The numbers in the centers are BLUE's expected gains, when he uses his optimum strategy. The answers are, of course, no better than the value judgments invoked in framing the assumptions.

COMPARISON OF OWN COURSES OF ACTION

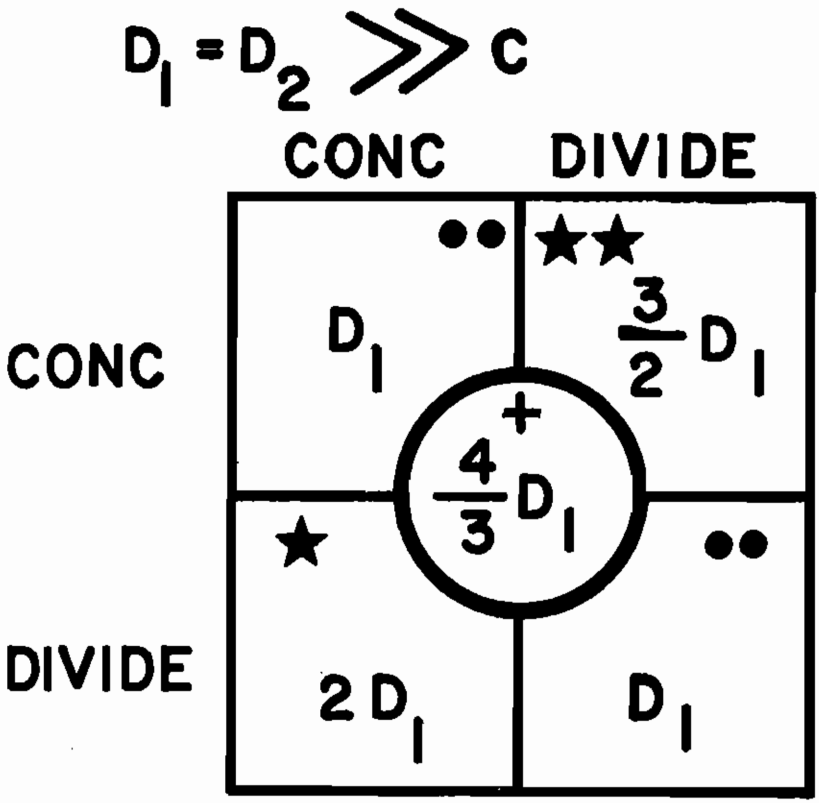

FIG. 9 A

COMPARISON OF OWN COURSES OF ACTION

$$2\,D_1 = D_2 \gg C$$

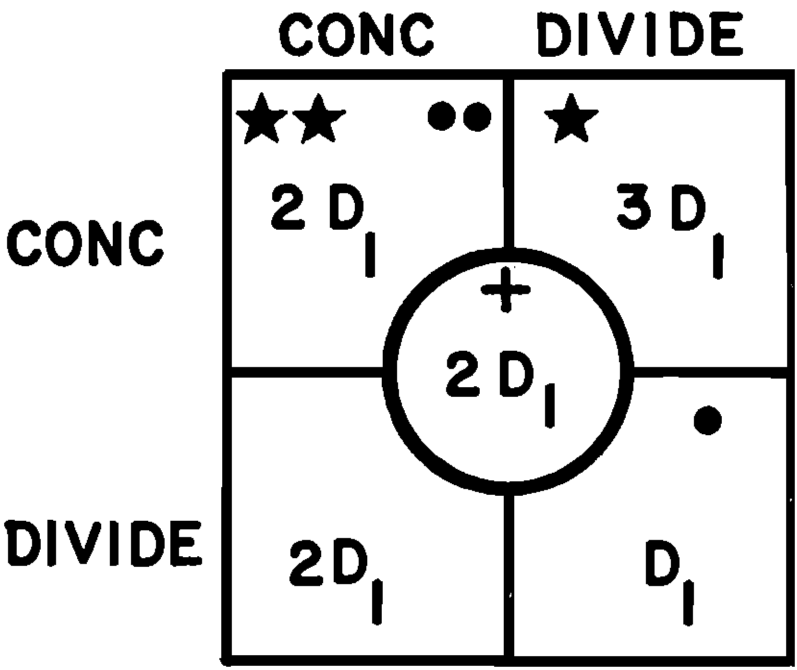

FIG. 9 B

COMPARISON OF OWN COURSES OF ACTION

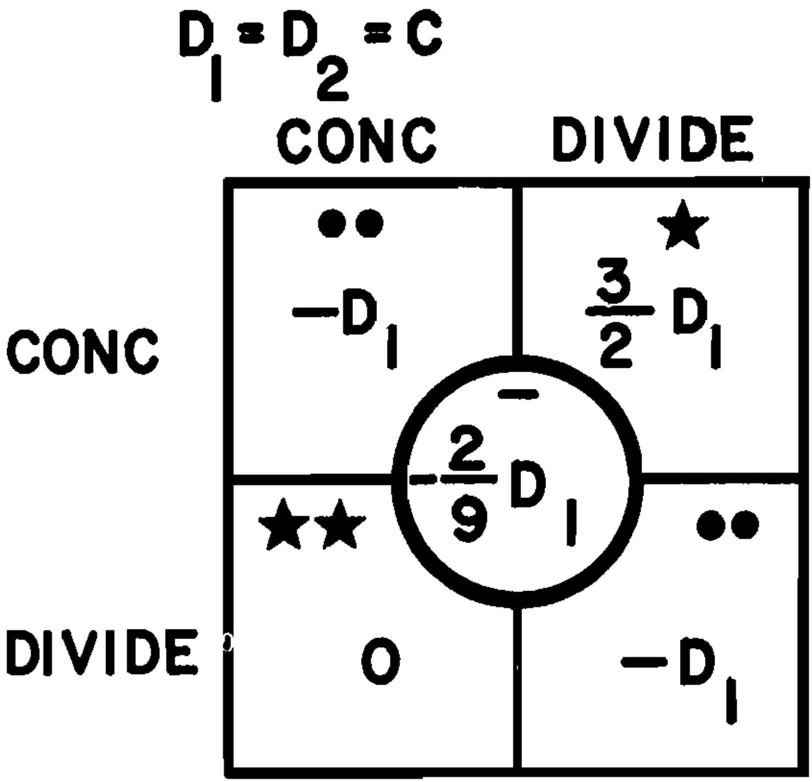

FIG. 9 C

In Figure 9a, D_1 and D_2 are the same and cruisers expendable, i.e., C is much smaller than D_1, ($C \ll D_1$). The maxmin and minmax are different.

The game theoretical *Decision* is as follows: BLUE should use a mixed strategy, determining by some random scheme whether to send the cruisers out singly or together. The best proportions are: together two-thirds of the time and separately one-third of the time. BLUE's expected pay-off is positive. If RED looks at the problem in the same way, he should also use a mixed strategy.

Figure 9b applies when D_2 is twice as large as D_1 and cruisers still expendable ($C \ll D_2$); the maxmin and minmax now become equal. Both sides should not concentrate their forces. BLUE's pay-off is still positive.

The solution changes drastically when BLUE places a high value on his cruisers, the "fleet-in-being" ($C \gg D_1$). This situation is shown in Figure 9c. His expected pay-off becomes negative, even with his optimum mixed strategy. The theory then suggests a radically different *Decision*. BLUE should stay in port and wait for a better opportunity.

F. Conclusion

Instead of reading a point-by-point summary I will close with some general remarks about the military implications of Game Theory.

Game Theory is concerned with the last step in the decision process, the selection of a course of action, or decision.

To do this some preliminaries are required. Possible opposing strategies have to be formulated consistent with the available resources. The probable outcomes for all interactions have to be assessed.

Hence we have to understand the rules of engagement. Outcomes have to be rated in order of preference. This requires a scale of military worth. This scale need not necessarily be quantitative but it must fit the problem at hand.

These prerequisite steps are inherent in every planning problem regardless of the method of solution. They are hard ones and demand the application of professional judgment of the highest order. Game Theory does not help us out of this box. For example, if we omit the winning strategy from the list we merely find a strategy that loses least.

Game Theory and the accepted military decision doctrine lead to identical answers when matched strategies exist. These are situations in which the same course of action is best against enemy capabilities or probable intentions.

The game theoretical approach and the traditional military planning process diverge when the matrix does not have a saddle-point. The game theoretical choice is now a mixed strategy. It can be regarded as a compromise between planning on capabilities and intentions. Following a mixed strategy rather than the safest pure one is a trade-off. It yields a higher expectation of gain at the risk of increasing the chances of an unfavorable outcome. This is still a conservative approach. It appears to be entirely appropriate for the stronger side, which will win as long as it can guide the course of the conflict as a whole in accordance with expectations. The weaker side cannot hope to win unless it adopts a more daring approach.

A mixed strategy only makes sense if the specific choice can be concealed until the enemy has made his move. The side that cannot avoid telegraphing its punches has to stick to strict capabilities planning and pay the price.

Game theoretical solutions are static. They can often be improved upon on the basis of new information, which becomes available as the action unfolds. There are no formal rules for this purpose at the present time.

Military problems are usually far too complicated for explicit solution. Here, Game Theory can still provide a general framework for decision and clarify key issues. But it is never a substitute for experience and good professional judgment.

Finally, a brief comment upon the relationship of Game Theory to War Gaming. Both try to come to grips with the same fundamental difficulty, the problem of enemy reaction. They tend to complement one another.

Game Theory can screen out unpromising strategies and find good mixtures for further investigation by gaming techniques.

War Gaming can help evolve a set of strategies for a game theoretical analysis. It can also give a better understanding of the rules of complex engagements.

Both may get us into trouble if inputs are in error or if the implicit value judgments are inappropriate for the real problem.

BIOGRAPHIC SKETCH

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Schools:

1940 - California Institute of Technology, B.S. Degree.
1941 - University of California (L.A.), M.A. Degree
1943 - Duke University, Ph.D. Degree.

Career Highlights:

1943-44 - Research Engineer, Am. Gas Assn.
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1946-52 - Research Chemist, Am. Cyanamid Co.
1952-date - Operations Evaluation Group, OpNav. assignments follow:

1953-54 - Assignments to CARDIV's 14 and 18.
1954-55 - Staff, COMANTISUBLANT.
1955-56 - Scientific Analyst, Submarine Branch, OpNav. (Op. 311)
1956 - Project NOBSKA.
1956-58 - Assistant to Director, NAVWAG (Op. 93R).
1958-59 - Staff, COMFIRSTFLT.
1959-61 - Staff, Naval War College.